Chapter 6.

Heat Conduction in Gas Turbine Airfoil

Determining temperature distributions within a gas turbine airfoil (blade or vane) with such complex geometry is important in estimating possible damage due to increasingly high temperatures used in modern engines. At steady state, the temperature distribution is governed by the Laplace equation, an elliptic partial differential equation. Complex flow outside the airfoil determines a boundary condition to the equation, and another complex flow of coolant inside the airfoil determines another boundary condition to the equation. Especially, the gas side surface condition has been extensively studied by many researchers in detail. Other studies have focused on the coolant flows, with various cooling schemes.

This chapter considers effects of conduction on temperature distributions in a turbine blade by analytical solutions and numerical solutions with and without geometrical curvature effects in the vicinity of the leading edge. The main focus is on comparisons between the above methods to see how the steep distribution in heat transfer coefficients near the leading edge on the gas side affects the solutions.

Unless otherwise noted, the parameters used in the analyses in this chapter are those summarized in Table 6.1
Table 6.1 Common parameters used in the conduction analyses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_g$</td>
<td>1200 [°C]</td>
<td>Gas side reference temperature</td>
</tr>
<tr>
<td>$T_{sg}$</td>
<td>700 [°C]</td>
<td>Gas side surface temperature</td>
</tr>
<tr>
<td>$h_g$</td>
<td>Davis data</td>
<td>Gas side heat transfer coefficients, provided by experiment (Baughn 1995)</td>
</tr>
<tr>
<td>$k$</td>
<td>20 [W/m-K]</td>
<td>Thermal conductivity of blade material</td>
</tr>
<tr>
<td>$T_c$</td>
<td>400 [°C]</td>
<td>Coolant side reference temperature</td>
</tr>
</tbody>
</table>

Figure 6.1 Airfoil cross-section (Baughn et al. 1995)
6.1 Geometry of Gas Turbine Airfoil

A cross-section of a gas turbine airfoil looks exactly like that of an airplane wing, except with a greater turning angle of the turbine airfoil. A sample turbine airfoil is shown in Fig. 6.1. The geometry varies from one airfoil to another. The shape of an airfoil is a factor that determines flow conditions around the airfoil. Therefore, to do a conduction analysis using data from previous experiments for which boundary conditions were measured, it is necessary to match the geometry of the airfoil between the experiment and the conduction analysis.
The airfoil shown in Fig. 6.1 is referred to here as the Langston blade. This blade shape also has a distribution of heat transfer coefficients available from an experiment as shown in Fig. 6.2 (Baughn et al. 1995). The present study works with this geometry, but there was concern about determining the thickness of the airfoil walls. The dashed line in Fig. 6.1 is not presented in the original source but is an estimated geometry assuming a uniform thickness of 1 [mm].

6.1.1 Scaling the airfoil to the actual size

The size of the airfoil used in Baughn et al. (1995) is not the actual size used in an engine, but the present conduction analysis is intended to be for an actual size. To scale down the Langston blade geometry presented in the paper, the chord length of a sample piece of a gas turbine blade was used. We have, in our laboratory, sample gas turbine blades which are believed to be very close to the Langston geometry. According to measurements on a sample blade cut in half at the middle of its span, the chord length is 30 [mm]. Figure 6.1 presents the scaled-down geometry of the Langston airfoil.

6.1.2 Estimating the wall thickness

The estimate of the wall thickness of 1 [mm] is made from the sample piece of a turbine blade. An optical comparator was used to trace the outside and the inside surfaces of the sample piece. To estimate the wall thickness of the Langston airfoil, the ratio of the outside radius of curvature to thickness, \( r_g/s \) at the leading edge of the sample piece was used. The values were taken at the leading edge because, for a value of the ratio less than a certain value, there would be no room for the inside geometry to have curvature near the leading edge. Without the inside curvature, it would be difficult to have an effective cooling scheme such as impingement or showerhead cooling. When the ratio was used to estimate the thickness, it was confirmed that the choice of 1.0 [mm] was satisfactory. Therefore, the thickness of 1.0 [mm] was set to be the standard value of thickness in the
following computations although it is varied in some cases to see the effects of the thickness.

### 6.1.3 Locating HTC distribution on the airfoil geometry

The heat transfer coefficient (HTC) distribution shown in Fig. 6.2 assumes a chord length of 10 [mm], converted following the discussion made in the section 6.1.1. To associate the geometry shown in Fig. 6.1 with this distribution, positions along the outside surface from the stagnation point, \( x_g = 0 \) in Fig. 6.2, were measured in Fig. 6.1. Then, the HTC distribution was located on the airfoil surface with the stagnation point as the reference point. These represent physical points in Fig. 6.1 for which there are corresponding HTC values in Fig. 5.2.

This wrapping process is important in the curved wall analysis in Section 6.4 because it determines the relative locations of the ridges and the steep gradients in the HTC distribution relative to the leading edge curvature, especially in the curved-wall-analyses section of this report.

### 6.1.4 Treatment of the trailing edge

When modeling an airfoil for a numerical calculation, the trailing edge must be considered. As observed in the sample blade, the inside geometry of such an airfoil can be complex with the surface being rougher than the outside, due to series of ribs connecting the pressure and the suction side walls and with coolant channels through the trailing region. All of these complexities are for enhanced blade-to-coolant heat transfer. They are difficult to be modeled. This modeling would not have been time-effective because little is known of the conditions of convective heat transfer coefficient on the coolant-side surface in the trailing edge region. For these analyses, the trailing edge region of the model was not carefully matched with the sample blade.

In the plane wall analysis, therefore, while the airfoil body is modeled as a wall with a uniform thickness, the trailing edge (or the two sides of the computational domain)
has a wrap-around boundary condition. This means that both temperature and flux match at both ends of the domain – identical points.

In the curved wall analysis, however, the modeled trailing edge has a zero-flux boundary condition. This is because, as seen in Fig. 5.2, the variation in HTC is quite flat at locations further away from the leading edge. The flatness in HTC distribution provides a condition of close to one-dimensional conduction, as shown in the next section of plane wall analysis. For faster computation, the domain length in the x-direction was shortened, the boundaries were assumed to have zero-flux.

6.2 Analytical Solutions

Solutions to one-dimensional analyses, i.e. heat conduction only in the direction of blade thickness, can be obtained analytically. In other words, the longitudinal conduction is not allowed. The results of the analytical solutions are compared with the numerical solutions in sections 6.3 and 6.4; therefore, only the developments and not the results of the 1-D analyses are discussed in this section.

6.2.1 Plane wall model

In this section, the blade is assumed to be a plane wall (Fig. 6.3, see page 87); no curvature effect is present. This model may be thought of as a hollow airfoil being stretched out. Since no longitudinal conduction is assumed, the Laplace equation reduces to:

\[
\frac{d^2 T_w}{dy^2} = 0 \tag{6.2.1}
\]

This can be directly integrated and yields:

\[
T_w(y) = c_1 y + c_2 \tag{6.2.2}
\]

The two boundary conditions are set at the gas side of the domain, and they are:

\[
T_w(0) = 700 \tag{6.2.3}
\]
Applying these to obtain the temperature distribution:

\[
\begin{align*}
\frac{dT_w}{dy}_{y=0} &= -\frac{h_g(T_g - T_w(0))}{k} \\
T_w(y) &= T_{sg,0} - \frac{h_g y}{k} \left( T_g - T_{sg,0} \right)
\end{align*}
\] (6.2.4)

The temperature difference between the gas side and the coolant side surfaces can be calculated by subtracting \( T_w(0) \) from \( T_w(s) \). This results in the following.

\[
\Delta T_{w,0} = \frac{s \cdot h_g}{k} \left( T_g - T_{sg,0} \right)
\] (6.2.5)

The bar on top of \( T \) designates plane wall. Because everything on the right hand side is constant over \( x \)-locations, except the heat transfer coefficient, \( h_g \), the distribution of the temperature difference, \( \Delta T_{w,0} \), is merely a linear function of the local heat transfer coefficients.

Now, when heat flux at a longitudinal position is considered, it is constant at any \( y \)-position since no curvature effect is present, i.e.

\[
q_{sg,0} = q_{sc,0} = q_0''
\] (6.2.7)

Thus, the values of the heat flux, \( q_0'' \), can be obtained as follows.

\[
q_0'' = h_g \left( T_g - T_{sg,0} \right)
\] (6.2.8)

Furthermore, by looking at Eqs. (5.7) and (5.8), the heat transfer coefficient distribution on the coolant side can be calculated.

\[
h_c = \frac{q_0''}{T_w(s) - T_c}
\] (6.2.9)

The above expressions are used in the section 5.2 to compare this one-dimensional analysis and the two-dimensional analysis.

### 6.2.2 Curved wall model

If the above plane-wall model were applied to the leading edge region where a strong curvature effect is present, the results would differ from reality. Since the leading edge
region has the highest heat transfer coefficients (highest heat fluxes) since the outside surface temperature is assumed to be uniform. This may result in high error.

To a heat flux coming into the blade wall from the gas side, a cylindrical section provides reduced area with decreasing radial distance thereby increasing the heat flux according to the ratio of areas available at different radii of curvature.

At the gas side surface, the heat flow through an infinitesimal angle, \( d\alpha \), can be expressed as follows.

\[
dQ = h_g \cdot (r_g \cdot d\alpha) \cdot (T_g - T_{sg})
\]

(6.2.10)

The same amount of the heat flow conducts through the wall.

\[
dQ = q'' \cdot (r \cdot d\alpha)
\]

(6.2.11)

The heat flux, \( q'' \), is determined by Fourier’s equation.

\[
q'' = -k \frac{dT_w}{dr}
\]

(6.2.12)

Combining and rearranging to get:

\[
dQ = \int_{\text{inside}}^{\text{outside}} \frac{1}{r} dr = k d\alpha \int_{\text{inside}}^{\text{outside}} dT
\]

(6.2.13)

Integration yields:

\[
dQ = \frac{kd\alpha}{\ln \left( \frac{r_g}{r_g - s} \right)} \cdot \Delta T_w
\]

(6.2.14)

Combining with (5.10) to get an expression for the temperature difference between the gas side and the coolant side.

\[
\Delta T_w = \frac{h_g \cdot r_g}{k} \ln \left( \frac{r_g}{r_g - s} \right) \left( T_g - T_{sg} \right)
\]

(6.2.15)

Here, \( \Delta T_w \) is curved-wall temperature difference. The corresponding expression for the plane wall is Eqn. (6.2.6). The ratio of temperature differences of the curved wall to those of the plane wall can be written as follows.
\[
\frac{\Delta \hat{T}_w}{\Delta \bar{T}_w} = \frac{r_g}{s} \cdot \ln \left( \frac{\frac{r_g}{s}}{\frac{r_g}{s} - 1} \right)
\]

(6.2.16)

This simple one-dimensional analysis shows that, for larger ratios, \(r_g/s\), the curved wall analysis is not necessary because the ratio of temperature differences in Eqn. (6.2.16) nears unity.

The analytical, one-dimensional calculations developed in this section are presented along with the results of numerical, two-dimensional calculations in subsequent sections to see differences between the analyses. Where there is little difference, one-dimensional results are sufficient.

### 6.3 Numerical Solutions Based on Plane Wall Assumption

#### 6.3.1 Problem formulation

In this section, the blade is assumed to be a plane wall with a uniform thickness. The computational domain and the corresponding physical domain are both flat and long rectangles. Therefore, much of the discussion made in the Section 6.1 does not apply. The discussion made in the Section 6.1.3 does not apply. Since there is no curvature, it does not matter where the HTC distribution is located relative to the physical domain.

The estimate of the wall thickness of 1 [mm] from the Section 5.1.2 can be thought of as a base case. Because the geometry is just a plane wall, the thickness can be varied to see the effects of wall thickness, which is less convenient to do with the curved wall analysis.

The boundary conditions to the domain are the tricky ones in this analysis. The trailing edge, the side boundaries in the computational domain, has a doubly connected boundary as discussed in the Section 6.1.4. The boundary conditions at the gas-side surface are both level and flux – temperature and heat flux. The heat flux is calculated based on the HTC distribution given in Fig. 6.2 while assuming a gas temperature of
1200 [$^\circ$C] and a surface temperature of 700 [$^\circ$C]. There is no boundary condition prescribed at the coolant-side surface. A goal of this computation is to determine the conditions at the coolant-side surface.

6.3.2 Procedures of computation

Since the problem prescribes two boundary conditions on the gas side of the domain, the problem must be solved as a parabolic system. As the typical parabolic system proceeds in time, the present computation proceeds from the gas side to the coolant side. Finite difference method of first order accuracy was used. The development of the finite difference equations and the solution technique are discussed in Chapter 5.
On the gas side (where \( j = JN \)), both flux and level (heat flux and temperature) are specified as boundary conditions. This makes the calculation for the level on the next layer \( (j = JN-1) \) possible. One may proceed inward from the gas side to the coolant side without the iteration that is usually necessary when solving an elliptic system.

A problem of this solution technique is that it generates, after a few layers of calculations, dispersion errors typically observed in parabolic systems. To overcome this problem, the level, or the temperature distribution on each layer is smoothed using a smoothing function to eliminate oscillations. The smoothed temperature distribution is used for the calculations of the level on the next layer. As noted above, the errors would become significant after several layers of calculations if there were no smoothing, but there would be no such significant errors after a single layer. In comparing the original and the smoothed profiles of temperature at a computational layer, differences between them are not apparent.

Once the stepwise calculations reach the coolant side, distributions of heat flux through the coolant-side surface are calculated based on the temperature gradients for various sizes of wall thickness. Wall thicknesses of 0, 0.5, 1.0, 1.5 and 2.0 [mm] are calculated. In fact, the case with 2.0-mm thickness provides the temperature distributions for all the cases. The distribution, for example, at 1.0 [mm] in the 2.0-mm case is the coolant-side distribution for the 1.0-mm case. Because the calculations are parabolic, the distribution at each layer depends only on the nodes in the direction of the gas side.

### 6.3.3 Results and discussions

Figure 6.4 presents the temperature distribution for the 2-mm-thickness domain. As noted above, the temperature profile at 1.0 [mm] would also be the one at the coolant-side surface if the thickness were 1 mm.

Figure 6.5 presents the temperature distributions on the coolant-side surface in the vicinity of the leading edge from the two- and the one-dimensional analyses. The one-dimensional analysis is done with Eqn. (6.2.5), assuming a thickness of 2 [mm].
Figure 6.4 Temperature contour of the plane-wall calculation with 2-mm wall thickness, temperature in [°C]
The temperature distributions in Fig. 6.5 have greater deviations from one another at the leading edge, and form almost an identical line further away from the leading edge. Since the one-dimensional analysis, Eqn. (6.2.6), is much simpler, the regions further away from the leading edge could have been done without numerical calculations. This is true also for the curved wall analysis in the next section.

One thing to note in Fig. 6.5 is that, at locations where the line of the two-dimensional analysis deviates from the one-dimensional analysis line, it does so in the way that makes the temperature gradients steeper. Thus, the lowest temperature in the domain, located on the coolant-side surface at the leading edge, is even lower for the two-dimensional analysis. This may be against one’s intuition which would assume that
increasing a dimension in such an analysis would smooth out and reduce the gradients. Considering a flat temperature profile to be the final form of a diffusion process. Imposing such a profile on the gas-side surface makes the coolant-side surface the beginning of the diffusion. However, since a heat flux is considered to forward in the direction of negative temperature gradient, perhaps this process should be called concentration of cooling. This point is discussed more in the Section 6.3.4.

Moreover, one can realize that this parabolic computational algorithm may be used to go backward in a diffusion process. Just as discussed above, having a flat temperature profile may be thought of as the beginning of a computation implies a computation proceeds in the direction opposite of the diffusion process.

![Figure 6.6](image-url)

Figure 6.6 Heat flux through the gas- and the coolant-side surfaces from two-dimensional, plane-wall analysis, using $k = 20$ [W/m-K]
Figure 6.6 presents the calculated heat flux distribution on both the gas- and the coolant-side surfaces. Again, at locations far away from the leading edge, there are few differences between the two sides, showing that the one-dimensional analysis may be satisfactory. The slight wiggling is probably due to computational instability.

Another issue in comparing heat fluxes on the two sides is related to conservation of energy. The area under each curve represents the total heat flow across each surface, and the areas must be one another in order for energy to be conserved. This point is discussed in the Section 6.3.4. Calculations confirm that energy is conserved.

The next two figures, Fig. 6.7 and 6.8, consider the effects of wall thickness. As discussed earlier, each line in the figure corresponds to both the profile on the coolant-side surface and the profile at that location in a 2-mm thick wall. Therefore, the discussions on the heat flux concentration still apply. In other words, since temperature gradients become steeper, the lowest temperature observed near the stagnation point becomes lower as the wall becomes thicker. Also, concentration of heat flux becomes more significant, and the heat flux near the stagnation point becomes higher as the wall becomes thicker.
Figure 6.7 Wall temperature at various distances from the gas-side surface (two-dimensional, plane-wall analysis)

Figure 6.8 Two-dimensional heat flux component in the y-direction at various distances from the gas-side surface
6.3.4 Validation of procedure

Many believe that a boundary condition must be known at each boundary and all around the domain in order to solve the elliptic Laplace equation. A question may arise whether it is possible to prescribe two boundary conditions at a boundary and none at another. Answer this requires testing the validity of the algorithm used in this computation. Four ways are identified.

First of all, conservation of energy can be tested. Since the domain is at steady state, the sum of all heat flow crossing the boundaries must be zero. Major heat flux is in the direction from the gas side to the coolant side. Moreover, since the side boundaries are doubly connected, there is no loss or gain of heat, or energy. Therefore, the heat flux through the gas-side surface must be equal to that through the coolant-side surface.

To calculate heat flow through each surface, the following formula can be used.

\[ Q = \int q'' \, dx \]  

(6.3.1)

Or, for the discretized computational domain, the integration must be converted to a summation, as follows.

\[ Q = \sum_{i=1}^{N} q''(i) dx(i) \]  

(6.3.2)

Since a uniform grid is used in this section, \( dx \) is not a function of the \( i \)-position. The calculations yield 34.9 [W/mm] on both the gas- and the coolant-side surfaces.

Secondly, a consistency analysis on the finite difference equation (FDE) examines whether the FDE is consistent with the original PDE. The details of the process are presented in Chapter 5. The analysis concludes that the finite difference equation developed for and used in this plane-wall analysis is consistent with the original Laplace PDE.

Thirdly, the gas-side HTCs are recalculated using an elliptic solver. The coolant-side temperature profile which is calculated from the parabolic solver is used as the boundary condition at the coolant side of the domain. The results allow the recomputed HTC’s to be compared against the original HTC distribution. Figure 6.9 compares the
original and the recovered (from this recalculation) HTC distributions. There is little
difference between the two lines, and they are undistinguishable in the one-dimensional
regions.

Lastly, looking at Fig. 6.4, one sees that the temperature distribution in the domain
is reasonable, considering the imposed boundary conditions. Near the gas side of
the domain, the temperature gradient in the y-direction is greater where there is greater
heat flux, i.e. near the leading edge. This forms dome-shaped contour lines that are
increasing in size toward the coolant side. Since the zero y-location represents also the
contour line of 700 [C], the flat line and the dome-shaped contour lines with increasing
size indicate concentration of heat flux on the inside surface toward the leading edge.

At the sides of the domain, or further away from the leading edge, the situations
are nearly one-dimensional. Temperature contours are relatively horizontal, indicating
little heat flux in the longitudinal direction (Fig. 6.4).
Figure 6.9 Comparison of the gas-side HTC distributions between the original and the recovered
6.4 Numerical Solutions Based on Curved Wall Assumption

6.4.1 Problem formulation

The boundary conditions are essentially the same as those of the previous section for the plane wall computation. Heat transfer coefficient distribution and a uniform temperature are prescribed on the gas side, and heat fluxes at the sides are set to zero.

The only difference from the plane wall computation is the leading edge geometry: a section of circular shell. Since the prescribed heat transfer coefficients were measured on the Langston blade geometry, the inside geometry is estimated from an available sample blade which is believed to be as close as the Langston blade. Following the measurement of the geometry, the eccentricity between the inside and the outside radii of curvature near the leading edge was measured, and this was applied to the computation. Therefore, a technique for generating computational nodes in this region was necessary. This is discussed in the following section.

6.4.2 Elliptic grid generator

Computational nodes that are uniformly distributed in a computational domain do not necessarily map to the corresponding physical domain if the physical domain is not a rectangle. In such a case, a technique using an elliptic partial differential equation can be used to uniformly distribute the computational nodes in the physical domain. The detailed procedures used here are recorded in Appendix E.

A theoretical background is discussed in Chapter 9 of Hoffman and Chiang (1998). The generator evenly distributes computational nodes over a computational domain, using an elliptic partial differential equation. In this way, one can avoid sudden changes in grid sizes, which can be a cause of computational errors. This method is helpful for domains for which distributing nodes evenly can be difficult, such as the eccentric cylindrical shell in the present study. The only disadvantage may be that the distances between nodes are no longer constant, but differ at every node. This slightly
complicates the computational routine and, more importantly, increases data storage necessary to run the routine. Nevertheless, the curved wall analysis has benefited much from this grid-generating algorithm. The generated grid is shown in Fig. E.2 in Appendix E.

6.4.3 Procedures of computation

The computational domain consists of three sections: two plane walls for the pressure and the suction sides and a circular shell for the leading edge. The physical length of the domain is shortened, compared to the plane wall case. This is because the regions close to the trailing edge on both suction and pressure sides are far enough to not be affected by the leading edge geometry. In these regions, the solution can be represented by the results from the plane wall computation, for which even one-dimensional analysis should represent. The thickness of the wall is assumed to be 1.0 [mm] and fixed in this analysis.

The problem prescribes two boundary conditions on the gas side of the domain. Thus, the problem is solved parabolicly from the gas side to the coolant side. This is the same procedure as in the plane wall case in the previous section. However, the side boundaries, toward the trailing edge, are set as zero heat flux. This is due to the shortened length of the domain. This can be justified by the fact that the effect of two-dimensional conduction has been determined to be small.

The algorithm is essentially the same as for the plane wall computation. The computation begins at the gas-side surface with prescribed temperature and heat flux determined from the distribution of HTCs (Fig. 6.2). It proceeds inward and reaches the coolant-side surface. Based on the temperature distribution at the coolant-side surface, the entire domain is again calculated, this time elliptically. Then, similarly to the plane wall analysis, the gas-side HTC distribution is recovered to be compared against the original, prescribed data. The heat flux from the coolant-side surface to the coolant, at the reference temperature of 400 [°C], is also calculated. The aim is at determining necessary heat transfer coefficients on the coolant side surface.
In addition, one-dimensional analytical calculation, discussed in Section 6.2.2, is also performed in the same algorithm (domain4.m, Appendix E).

### 6.4.4 Results and discussion

Figure 6.10 shows the resulting temperature distribution over the domain. It shows also the curved wall model. Since the direction of heat flux is perpendicular to isothermal lines, the figure shows extreme concentration of heat flux toward the back of the leading edge. Figure 6.10 can be compared with the plane wall case, Fig. 6.4. First, the lowest temperature occurs at the same point, i.e. at \( y = 1.0 \text{ [mm]} \). However, it is approximately 480 [°C] for the curved wall case (Fig. 6.10) and lower than the plane wall case having 600 [°C] (Fig. 6.4). Since the gas-side conditions (temperature and heat flux) are the same for both cases, this difference in the extreme temperature shows a great effect of leading edge curvature.

![Figure 6.10: Temperature distribution, curved wall model](image-url)
Figure 6.11 presents temperature profiles at the coolant-side surface, comparing the one-dimensional calculation (Eqn. 6.2.15) and the two-dimensional computation. Equivalent to this in the plane wall analysis is Fig. 6.5. However, the difference is that the one-dimensional solution is based on Eqn. (6.2.6). The deviation of the 1-D analysis from the corresponding 2-D analysis is greater in the curved wall analysis than in the plane wall analysis. Again, the deviation occurs in the way that the temperature gradients are greater in the 2-D analysis than in the 1-D analysis.

The greater deviation from the 1-D solution in this case may be explained by the eccentricity. It is not considered in the one-dimensional analysis but is incorporated in the 2-D solution. The eccentricity increases the thickness of the model wall by 50%, making it 1.5 [mm] at the leading edge. This makes the heat flux concentrate more in the two-dimensional analysis. Hence, the greater deviation is expected for this analysis.

Figure 6.11: Coolant-side surface temperature after curved-wall analysis, comparison of 2-D and 1-D analyses
Figure 6.12 presents a comparison of heat flux crossing the gas- and coolant-side surfaces for the curved-wall analysis. The corresponding plot for the plane wall solution is Fig. 6.6. There is a tremendous difference in the maximum local heat flux at the leading edge; it is about 2.1 [W/mm²] for the plane wall solution while it is over 14 [W/mm²] for the curved wall solution. Greater concentration of heat flux takes place in the curved wall solution such that the gas-side profile of heat flux does not compare with the coolant-side profile. Furthermore, the level of heat flux near the leading edge on the coolant side is an order greater than for the rest of the airfoil. Even on the coolant side, the essentially one-dimensional region has fluxes less than 1.0 [W/mm²].

A low peak in the coolant-side profile near $x_c = -0.5$ is thought to be an error in the computation due to the junction of a plane wall and the circular shell. At this point, the grid spacing is not uniform since the plane-wall domains are not included in the grid-generation algorithm.

![Graph showing heat flux through gas- and coolant-side surfaces](image-url)

**Figure 6.12**: Heat flux through the gas- and coolant-side surfaces, after two-dimensional, curved wall analysis, using $k = 20$ [W/m-K]
A major concern, however, has risen with this heat flux profile at the coolant surface: negative heat transfer coefficients. They occur at the sides of the leading edge, where the heat flux is negative (Fig. 6.12). The negative heat flux indicates the direction of heat flow from the coolant to the surface. However, as seen in Fig. 6.10, the temperature at the coolant surface is greater than that of the coolant everywhere. These two figures then imply negative heat transfer coefficients at locations where the heat fluxes are negative.

The extreme geometry is considered to be a cause of these conditions. The plane wall solution does not result in negative values of heat flux or heat transfer coefficients. However, Fig. 6.6 shows that coolant-side heat fluxes decrease at the sides of the leading edge. Therefore, the mechanism that decreases heat fluxes is the same as the one seen in the plane wall analysis. The only difference in this curved wall analysis is the cylindrical geometry at the leading edge. The cylindrical geometry has necessitated extreme concentration of heat flux to fulfill the given boundary conditions such that heat transfer coefficients are to be negative at some locations. Certainly, this is unrealizable in an actual gas turbine.

Therefore, in comparing the curved-wall analysis with the plane wall one, one sees that it is clear that the curvature at the leading edge should not be neglected. This is because of the very steep gradients in the profile of heat fluxes on the gas side and a peak value that is much higher than at any other location. The initial goal of this heat conduction problem is to determine coolant-side conditions from given gas-side conditions. However, further studies are necessary to incorporate the curvature effects of the airfoil leading edge. Possible development in the future may include allowing higher temperatures at the gas-side leading edge to ease the concentration of heat flux. Also, potential HTC distribution at the coolant side could be prescribed so that the problem could be solved elliptically.
6.5 Conclusions

Heat conduction in modeled gas turbine airfoils are examined. The study has evolved from a previous study by Eckert et al. (1997). The present study is aimed at determining conditions at the coolant-side surface of model gas turbine airfoils by prescribing the gas-side heat transfer coefficients and uniform temperature. Both analytical and numerical methods are employed to solve the problem one- and two-dimensionally. The models include a plane wall that stretches the pressure and suction sides of an airfoil and a curved wall that takes the leading-edge curvature into account.

Analytical solutions assuming one-dimensional conduction from the gas side to the coolant side are developed for both a plane wall and a curved wall. The results are expressed in terms of temperature differences between the gas-side and the coolant-side surfaces. Equation (6.2.6) is for the plane wall while Eqn. (6.2.15) is for the curved wall. Both these solutions are plotted along with the corresponding two-dimensional, numerical solutions.

Numerical solutions which assume two-dimensional conduction are developed for a plane wall and a curved wall. The results are presented in terms of temperature distributions in the entire domains (Figs. 6.4 and 6.10), temperature profiles at the coolant-side surface (Figs. 6.5 and 6.11) and heat flux profiles at the coolant side compared with those of the gas side (Figs. 6.6 and 6.12). For the computations, the parabolic solution algorithm that is developed in Chapter 5 is used. This algorithm worked well in the plane wall calculation, as verified by the recovered HTC distribution at the gas side. The curved wall calculation has yielded negative heat transfer coefficients at some locations. Thus, further studies on the curvature effect are suggested.

In general, two-dimensional analyses are necessary in the vicinity of the leading edge because of the greatest gradients in the HTC distribution on the gas side. Also, the curvature of the leading edge should have significant effects that may be completely underestimated by one-dimensional solutions. In addition, future studies should be done to model the effects of ribs and trailing-edge geometry, present in a real airfoil.