Financial Analysis Primer

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Overview

• Time value of money
• Interest formulas
• Project evaluations
• Inflation and CPI
• Financial risk and financing

Time Value of Money

~ Today is worth more than tomorrow ~

Lender Makes an Investment

• Letting somebody use your money at a given time.
• Loses his/her purchase power for that time.
• Gets compensated when the money is returned at a later time.

Borrower Borrows

• Can consume product/service using somebody else’s $.
• Gains his/her purchase power for that time.
• Needs to compensate the lender when the money is returned at a later time.

Time Value of Money

• Comparison:
  --invest today’s $1 today
  --receive tomorrow’s $1 tomorrow
• Your money loses purchase power over time due to inflation.
• Today is worth more than future.
• Tomorrow is available only if today is survived.
**Interest Formulas**

~ “Present value” of a project ~

**Interest Rate**

- A rate of change of a $ value.
- Other rates:
  - Discount rate
  - Inflation rate
  - (Internal) rate of return

**“Present Value”**

- Simple example:
  - What is the present value, \( P \), of next year’s $11 (=F), given \( r=10\% \)?
  
  \[
  P = \frac{11\$}{1 + 0.1} = 10\$
  \]

**“Present Value” in general**

- What is the present value, \( P \), of $ in the future, \( F \), that is \( n \) periods away, given \( r \)?

  \[
  P = \frac{F}{(1+r)^n}
  \]

**Cash Flow Diagram**

- To calculate present value
- Depend on types of:
  - payments
    - single, equal and gradient
  - interest compounding
    - discrete (and continuous)

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1) Single Payment

\[ P = \frac{F}{(1+r)^n} \]

r: interest rate

2) Equal Payments

• Equivalent of equal future payments

\[ P = \frac{(1+r)^n - 1}{r(1+r)^n} A \]

r: interest rate

3) Gradient Payments 1

• Uniform gradient

\[ P = \left[ 1 - \frac{n}{r (1+r)^n - 1} \right] G \]

r: interest rate

4) Gradient Payments 2

• Geometric gradient

\[ P = \left[ \frac{(1+g')^n - 1}{g'(1+g')^n} \right] \frac{F_1}{1+g} \]

where \( g' = \frac{1+r}{1+g} - 1 \)

5) Continuous Compounding

• Uniform gradient

\[ P = \left[ 1 - \frac{e^r - 1}{e^{rn} - 1} \right] G \]

r: interest rate

6) In Reality ...

• Cash flow are more complicated:
**Project Evaluations**

~ Comparing Alternatives ~

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**Some Techniques**

- Net Present Value (NPV)
- Benefit-Cost ratio
- Internal Rate of Return (IRR)
- Payback period
- Capitalized equivalent
- Capital recovery with return
- Project balance

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**Net Present Value, NPV**

- Sum of the present value of net cash flow in each of the future periods

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**Calculating NPV**

- Equivalent present value of a project

\[
NPV = \sum_{t=0}^{n} \frac{R_t - C_t}{(1+r)^t}
\]

- Others: Annual equivalent and net future value

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**Benefit-Cost Ratio, BCR**

- Ratio of present-value benefit to present-value cost

\[
\frac{PB}{PC}
\]
**Internal Rate of Return, IRR**

- Interest rate that makes NPV zero.
  - Choose a project with the lowest IRR.
    - For a project that requires advanced investment

\[
NPV = 0 = \sum_{t=0}^{n} \frac{R_t - C_t}{(1 + i^*)^t}
\]

where \(i^*\) is IRR

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**Implications to Public Projects**

- Financial analysis becomes economic analysis.
- Non-monetary benefits must be included, as well as non-monetary costs.
- BCR analysis becomes tricky!
  - Some costs may be considered negative benefits, and vice versa.
  - NPV and IRR methods work fine.

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**Inflation and CPI**

- ~real interest rate~

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**Consumer Price Index, CPI**

- Price index of retail goods/services for a given year.
- Price index is a ratio of a price in a given year to that in the base year.

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**Inflation**

Rate of increase in CPI over the period of interest

For year \(t\), the inflation rate, \(f\), is calculated by:

\[
 f_t = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}
\]

where CPI is determined at the end of each year.

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**Real Interest Rate, \(r'\)**

- Also called:
  - inflation-free interest rate
  - constant-dollar interest rate

\[
r' = \frac{1 + r}{1 + f} - 1
\]

- The nominal interest rate, \(r\), must be greater than inflation rate, \(f\), in order for a positive real interest rate.
**Financial Risks**

~ Risk Management in Project Financing ~

**What is risk?**

- Probability of an adverse effect
  \[ \text{Risk} = \text{Magnitude (outcome)} \times \text{Probability} \]

- Hazard x Exposure (in public health)

**Uncertainty**

- Uncertainty about future
- Range of possible values
  - e.g. Range of possible cost for year t
- Complicates project evaluations
- Sensitivity analysis may be used.

**“Expected” Outcome**

- Terminology from statistics
- The mean of outcome
- Allows separation of
  - alternative evaluations and
  - risk comparisons
- Risk associated with uncertainty

**NPV of a Project with Risk**

An example:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>NPV [$]</td>
<td>-30,000</td>
<td>10,000</td>
<td>100</td>
</tr>
</tbody>
</table>

Expected NPV of the project, \( E \):

\[
E(\text{outcome}) = 0.1NPV_A + 0.6NPV_B + 0.3NPV_C \\
= -3,000 + 6,000 + 30 \\
= 3,030
\]
Other Techniques

- Sensitivity analysis
- Expected variance
- Monte Carlo analysis
- Decision Trees
- Payoff Matrix
- Maximin/Maximax Rules
- Hurwicz Rule
- Minimax Regret Rule

End of Financial Analysis