Self-Sensing Dual Push-Pull Solenoids using a Finite Dimension Flux-observer

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Abstract—Position feedback in a solenoid actuated system typically requires a position sensing device such as a Linear Variable Differential Transformer (LVDT). The goal of self-sensing is to obtain position information directly from the electrical signals to the solenoid actuators, thus obviating the additional cost and footprint of a LVDT or another displacement sensing device. Such measurement is possible due to the position dependence of electrical inductance in the solenoids. This paper proposes a finite-dimensional nonlinear observer for the magnetic flux linkage for the solenoids. Once the flux linkage has been identified, the solenoid position can be determined via the position-inductance relationship. The algorithm has been adapted for actual solenoids modeled as a third-order system that includes two eddy current modes accurate up to 1024 Hz. Implementation on commercially low-cost solenoids (with 5mm stroke) has demonstrated RMS position accuracy up to 0.061mm. The ability to self-sense accurately can enable solenoids to be deployed at cost for many motion control applications besides hydraulic valves.

I. INTRODUCTION

Solenoids are low-cost actuators but position feedback is required to utilize them for position control. The cost for position sensors such as Linear Variable Differential Transformers (LVDT), however, can be prohibitive. Self-sensing refers to extracting position information from the inherent signals like currents and voltages. This technique has attracted the attention of researchers in fields such as magnetic bearing for heart pumps in order to minimize the number of wires [1], or self-sensing switched reluctance motors, which is the cost effective and robust solution for electric motors [2], [4]. Self-sensing solenoids will also be useful to reduce the cost of high performance electro-hydraulic valves which are typically solenoid actuated but currently use LDVT for spool position feedback.

In this paper, we focus on dual solenoid actuators in the push-pull configuration in which the solenoids are mechanically connected together (see Fig. 1). As one solenoid retracts the other extends. Since solenoids produce only force in one direction, the alternative is a single solenoid with a spring return. This requires only one solenoid but has a lower force capability and needs a bias current to overcome the spring force so as to stay in place.

A finite dimensional nonlinear flux linkage observer is proposed to estimate the position information of a pair of push-pull solenoids. This is based upon the position dependent inductance characteristic of the solenoids. The observer design combines the inductance information from both solenoids to form a nonlinear time varying output equation. Once the flux is estimated, the position can be obtained from the position-inductance relationships.

Our earlier approaches to self-sensing in [6], [7] also focused on the dual solenoids in push-pull configuration. The approach in [6] explicitly solves for the initial flux linkage over a finite moving time horizon and utilizes several time delayed (hence infinite dimensional) filters. Consequently, it is rather complicated and is susceptible to singularity. The approach in [7] advances the self-sensing problem as an observer problem but still relies on an infinite dimensional boxcar filter. In the present paper, a nonlinear time-varying output equation is formulated which allows us to cast the problem as a finite dimensional observer problem. The Kresselmeier estimator [11] originally developed for estimating constant unknown parameters is adapted for use as an observer. The observer has some unique properties such as an forgetting factor and direct access to the observer error that can be useful.

Other self-sensing approaches for solenoids include [8] which uses the position dependent phase information at a resonant frequency signal to detect position. In [9], the position dependence of the rate of current rise with PWM input is used to estimate position. In both cases, the specific input wave-forms are assumed and the detection algorithms do not account for the motion induced variation in the current. In [10], a nonlinear observer is designed for a solenoid system that includes eddy current dynamics. Both mechanical and electrical aspects are used in the design. In contrast to these works, our proposed flux observer allows for arbitrary voltage waveforms and fully takes into account the motion induced current variation. In addition, the flux linkage observer primarily utilizes the electrical characteristics and does not require a model of the mechanical system.

The proposed flux observer is experimentally tested on a pair of low cost solenoids. System identification reveals
the solenoids to be 3rd order systems which incorporate two modes of eddy current dynamics. To apply the flux observer, the 3rd order solenoid system is first converted into a first order inductance model by applying a pre-feedback to compensate for the eddy current effect. RMS position estimation accuracy of 0.061 mm (over a stroke of 5mm) has been achieved.

The rest of paper is organized as follows. In section II, we formulate the model of a dual-solenoid actuator. Section III presents the principle and the analysis of the observer. Some experimental results are presented in Section V. Section VI contains concluding remarks.

II. SYSTEM MODELS

A. Ideal dynamics

We consider two ideal solenoids connected in an opposing push-pull manner (Fig. 1). Suppose \( x = 0 \) corresponds to the actuator being at one extreme position. The position dependent inductances of the two solenoids are modeled as:

\[
L_1(x) = \frac{\beta_1}{d_1 + x} - \bar{L}_1, \quad L_2(x) = \frac{\beta_2}{d_2 - x} - \bar{L}_2
\]

Let \( V_1, V_2 \) be the voltage inputs and \( i_1, i_2 \) be the currents going through the two inductances. The flux linkages for the two solenoids are given by:

\[
\lambda_1 = L_1(x)i_1, \quad \lambda_2 = L_2(x)i_2
\]

An idealized model of a solenoid is a L-R circuit (Fig. 2). Therefore, the dynamics for the ideal push-pull solenoids are:

\[
\begin{align*}
\dot{\lambda}_1 &= -R_{s1}i_1 + V_1 \\
\dot{\lambda}_2 &= -R_{s2}i_2 + V_2
\end{align*}
\]

Assuming that \( V_1, V_2 \) and \( i_1, i_2 \) can be measured, our goal is to estimate the flux linkages \( \lambda_1(t) \) and \( \lambda_2(t) \), and then from using the measured currents \( i_1(t), i_2(t) \) and inductance models \( L_1(x), L_2(x) \) in (1) to determine position \( x(t) \). Note that this enterprise does not rely on models of the mechanical aspect of the solenoids.

B. Eddy current dynamics

The model in (2)-(3) would suggest that the frequency response (with voltage input and current output) at fixed positions will be that of a first order system

\[
I(s) = \frac{1}{L(x)s + R_s}V(s)
\]

Detailed frequency domain identification (see Section V) however suggests that a third order model is needed to accurately model the dynamics up to 1 khz. To explain the third order dynamics, a 2 mode eddy current model (for each solenoid) is proposed:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix}
L(x) & \begin{pmatrix} 1 & k_1 & k_2 \\ k_2 & 0 & 1 \end{pmatrix} & \begin{bmatrix} i \\ i_{e1} \\ i_{e2} \end{bmatrix} \\
0 & 0 & 0 \\
0 & -R_{e1} & 0 \\
0 & 0 & -R_{e2} \\
\end{bmatrix} & \begin{bmatrix} i \\ i_{e1} \\ i_{e2} \end{bmatrix} + \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

Here, \( i_{e1} \) and \( i_{e2} \) represent the eddy currents induced by the primary current \( i \), \( k_1 \) and \( k_2 \) are respectively the mutual inductance coefficients, and \( R_{e1} \) and \( R_{e2} \) are the resistances of the eddy current loops. A useful way to visualize the eddy current effect is that of an AC transformer in which the primary coil and the two secondary coils are wound around a common iron core (Fig. 3). In this way, changes in the air gap (via \( x \)) would change the inductances of both the primary and secondary coils. This hypothesis is supported by the fact that frequency responses at different locations collapse to a common frequency response with appropriate \( 1/L(x) \) scaling of the frequencies. The mutual inductance coefficients \( k_1 \) and \( k_2 \) as well as \( L(x) \) were experimentally identified from the frequency response data. Although it would have been more complete to include the mutual inductance term between the the 2 eddy current loops, it is found not to improve the fit with the experimental data.

Denote

\[
K^{-1} = \frac{1}{g} \begin{pmatrix} g^T \\ G \end{pmatrix} / \det(K)
\]
where $K$ is defined in (4), $g \in \mathbb{R}^{1 \times 2}$, $G \in \mathbb{R}^{2 \times 2}$. Multiplying $K^{-1}$ on both sides of (4), we can see that the eddy current is equivalent to the simple first order model in (3):

$$\dot{\lambda} = \frac{d}{dt}[L(x)i] = F[V - Rsi]$$

(5)

where $F[u]$ is the output of the second order system:

$$\dot{\lambda}_E = - \frac{GR_E}{L(x)} \lambda_E + g \cdot u$$

$$y = \left(-\frac{g^T R_E}{L(x)(t)} \lambda_E + u\right)/\det(K)$$

(6)

where $R_E = \text{diag}(R_{t1}, R_{t2})$. Here, $\lambda_E = (L(x) i_{x1}, L(x) i_{x2})^T$ are in fact the flux linkages of the eddy currents. It can be shown that $\dot{F}[V - Rsi]$ is a low pass filtered version of $(V - Rsi)$. Although (6) is time-varying and requires knowledge of $x$, the bandwidths of filters are quite high so that using an estimate of $x$ seems adequate as long as $V(t) - Rsi(t)$ does not have significant high frequency content.

C. Nonlinear output equation

For the state equations (3), $i_1(t) = \lambda_1(t)/L_1(x)$ and $i_2(t) = \lambda_2(t)/L_2(x)$ can be considered output equations. If $L_1(x)$ and $L_2(x)$ are known, then the observer design is quite trivial. The difficulty lies in the dependence on the actuator displacement, $x$, being unknown.

With the push-pull solenoid configuration, the two flux linkage equations in (2) can be combined to form a new nonlinear output equation that does not depend on $x$. By eliminating $x$ from the inductances in (1), an implicit relationship between the two inductances can be obtained:

$$\frac{\beta_1}{L_1(x) - \bar{L}_1} + \frac{\beta_2}{d_1 + x} = d_1 + d_2 =: \bar{d}$$

(7)

Re-arranging, we have:

$$-L_1(x) L_2(x) + aL_1(x) + bL_2(x) + c = 0.$$  

where

$$a := -(L_2 + \beta_2/\bar{d})$$

$$b := -(L_1 + \beta_1/\bar{d})$$

$$c := (L_2 \beta_1 + L_1 \beta_2)/\bar{d} + \bar{L}_1 \bar{L}_2$$

Substituting

$$\lambda_1(t) = L_1(x) i_{1}(t); \quad \lambda_2(t) = L_2(x) i_{2}(t)$$

into (7) and rearranging, we have

$$0 = h(t, \lambda_1, \lambda_2) := a \cdot i_{2}(t) \lambda_1 + b \cdot i_{1}(t) \lambda_2 + \lambda_1 \lambda_2 + c \cdot i_{1}(t) i_{2}(t)$$

(8)

so that the dependence on $x$ has now been eliminated. Eq. (8) will be treated as a state varying output equation of the system (with output identically 0). Assuming observability condition holds, our approach is to design an observer to estimate $\lambda_1(t)$ and $\lambda_2(t)$ based on output $h(t, \lambda_1(t), \lambda_2(t))$.

In particular, if the flux estimates $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are incorrect, $h(t, \hat{\lambda}_1, \hat{\lambda}_2)$ will be non-zero and will provide error for adjusting the estimates.

III. Flux observer designs

The general approach is to utilize the nonlinear output equation (8) as feedback for an observer for the flux $(\lambda_1, \lambda_2)$. With the estimate of $(\lambda_1, \lambda_2)$, the actuator displacement $x$ is obtained from the inductance relationships (2) and (1). This section presents the flux linkage observer. Recovery of the actuator displacement $x$ will be presented in section IV.

A. General structure

Since the right hand side of (3) (1st order) or (5) (3rd order) is known (either as filtered version of the input and measurement), we consider a flux observer structure of the form:

$$\dot{\hat{\lambda}}_1 = \overline{F}[V_1 - R_{t1} i_1] + w_1(t)$$

$$\dot{\hat{\lambda}}_2 = \overline{F}[V_2 - R_{t2} i_2] + w_2(t)$$

(9)

where $w(t) = [w_1(t), w_2(t)]^T$ is the output injection to be designed based on the output prediction error:

$$e := h(t, \hat{\lambda}_1, \hat{\lambda}_2) = a \cdot i_2(t) \hat{\lambda}_1 + b \cdot i_1(t) \hat{\lambda}_2 - \hat{\lambda}_1 \hat{\lambda}_2 + c \cdot i_1(t) i_2(t)$$

(10)

Using the notation,

$$\hat{\lambda} := [\hat{\lambda}_1, \hat{\lambda}_2]^T$$

and subtracting $h(t, \lambda_1, \lambda_2) = 0$ in (8), (9) can be written exactly as:

$$e = \phi(t) \hat{\lambda} - \lambda_1 \lambda_2$$

where $\phi(t)$ is a measurable regressor given by:

$$\phi(t) := [a \cdot i_2(t) + \lambda_2(t), b \cdot i_1(t) + \lambda_1(t)]$$

(11)

The flux estimation error dynamics are:

$$\dot{\hat{\lambda}} = w(t); \quad \hat{\lambda}(0) = \lambda_0$$

(12)

$$e = \phi(t) \lambda(t) - \hat{\lambda}_1 \lambda_2(t)$$

(13)

B. Kresselmeier Estimator

Here, the exponential observer first proposed by Kresselmeier in [11] is adapted for the flux observer.

The goal is to construct the output injection to be similar to:

$$w(t) = -\gamma R(t) \hat{\lambda}$$

where $R(t)$ is a positive definite matrix. If this is possible, we would have the stable error dynamics:

$$\dot{\hat{\lambda}} = -R(t) \hat{\lambda}$$

The obvious challenge is that $\hat{\lambda}$ is unknown.

The key to the Kresselmeier method is a set of low pass filters of the regressor and of the output error:

$$\dot{\hat{R}}(t) = -q \cdot (R(t) - \phi^T(t) \phi(t))$$

(14)

$$\dot{\hat{p}}(t) = -q \cdot (p(t) - \phi^T(t) e(t)) + R(t) w(t)$$

(15)

where $q > 0$ is the filter’s bandwidth. For simplicity, assume that these filters have zero initial conditions. The output injection is then defined as:

$$w = -\gamma p$$

(16)
Lemma 1 With the filters in (16),
\[ p(t) = R(t) \dot{\lambda} + p_e(t) \] (17)
where \( p_e(t) \) is the output of the filter equation:
\[ \dot{p}_e = -q(p_e - \phi^T(t)\dot{\lambda}_1 \dot{\lambda}_2) \] (18)
Therefore without \( p_e \), we can use \( p \) to achieve of desired injection. The presence of \( p_e \) in (17) is due to the \( \dot{\lambda}_1 \dot{\lambda}_2 \) term in the output equation (13).

Proof: Both sides of (17) have the same zero initial conditions. Differentiating both sides, we see that both sides satisfy the same linear differential equation:
\[ \dot{X} = -q \cdot \left( X - \phi^T(t)(e(t) + \dot{\lambda}_1 \dot{\lambda}_2) \right) + R(t)w(t) \]
By the existence and uniqueness property of linear differential equations, both sides of (17) are the same. \( \Diamond \)

Remarks:
1) \( R(t) \) is the filtered version of \( \phi(t)^T \phi(t) \). While \( \phi(t) \) is seldom full rank, the collection of \( \phi(t)^T \phi(t) \) over time can be. The filter gain \( q \) controls the time constant and can be thought of as a forgetting factor.
2) If \( R(t) \) becomes full rank at some time \( t_0 \) it remains full rank for all \( t > t_0 \) although its co-norm may become smaller and smaller.
3) In the absence of the nonlinear term in (13), when \( R(t) \) is full rank, the observer error \( \lambda \) can be obtained from \( \lambda = R^{-1}(t)p(t) \). This is unlike other estimator or observer.

Let \( r(t) = \phi(R(t)) \) be the minimum eigenvalue of \( R(t) \) such that for all \( x \in \mathbb{R}^n \)
\[ x^T R(t)x \geq r(t)\|x\|_2^2 \]
We assume that the regressor \( \phi(x) \) is sufficiently exciting such that \( 0 < \varepsilon \leq r(t) \). In practice, this can be achieved by superimposing on any input to the solenoid a persistent high frequency excitation signal.

Since \( p(t) = R(t)\lambda(t) + p_e \), we have the dynamics:
\[ \dot{\lambda} = -\gamma R(t)\dot{\lambda} - \gamma p_e \]
\[ \dot{p}_e = -q \cdot (p_e - \phi^T(t)\dot{\lambda}_1 \dot{\lambda}_2) \]
Using a Lyapunov function
\[ W = \frac{1}{2} \dot{\lambda}^T \dot{\lambda} + \frac{1}{2} \dot{p}_e^T \dot{p}_e \]
\[ \dot{W} = -\dot{\lambda}^T p_e (\phi(t) - \phi^T(t)\dot{\lambda}_1 \dot{\lambda}_2) + \dot{p}_e^T \left( -\gamma R(t) \gamma \lambda + \gamma \dot{\lambda}^T \dot{\lambda}_1 \dot{\lambda}_2 \right) \]
\[ = -\frac{1}{2} \dot{\lambda}^T \left( \begin{array}{c} \gamma R(t) \\ -\gamma R(t) \end{array} \right) \left( \begin{array}{c} \dot{\lambda}^T \\ 0 \end{array} \right) \]
\[ = -\left( \lambda^T \right) p_e^T \left( \begin{array}{c} \gamma R(t) \\ -\gamma R(t) \end{array} \right) \frac{\dot{\lambda}^T}{\dot{\lambda}} \]
(19)
where
\[ s(t, \dot{\lambda}) = \frac{\gamma}{2} \dot{\lambda} \]
so that \( \|s(t, \dot{\lambda})\| \rightarrow 0 \) and \( \dot{\lambda} \rightarrow 0 \). Therefore, in the neighborhood of \( \dot{\lambda} = 0 \), the matrix in (19) is positive definite if \( \gamma < 4r(t)q \) so that \( (\dot{\lambda}, p_e) = (0, 0) \) is asymptotically stable.

IV. ACTUATOR DISPLACEMENT ESTIMATION

With the flux linkage estimates \( \dot{\lambda}_1(t) \) and \( \dot{\lambda}_2(t) \), we now estimate the solenoid position \( x \). Expanding (1) and assuming the estimated fluxes are the actual fluxes in (2):
\[ (d_1 + x)(\dot{\lambda}_1 + L_1 i_1(t)) = \beta_1 i_1(t) \] (20)
\[ (d_2 - x)(\dot{\lambda}_2 + L_2 i_2(t)) = \beta_2 i_2(t) \] (21)
Rearranging, we have
\[ (\beta_1 i_1(t) - d_1(\dot{\lambda}_1 + L_1 i_1(t))) = \frac{1}{2} x(t) \] (22)
\[ y_x(t) \]
While \( x(t) \) can be obtained using a least square algorithm using the instantaneous estimate \( \dot{\lambda} \), we design an observer to provide some temporal averaging.

By assuming the displacement dynamics,
\[ \dot{x} = v(t) \]
where velocity \( v(t) \) is a zero mean random process, the observer for \( x \) is:
\[ \dot{x} = -\lambda_x \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) \dot{x} - y_x(t) \]
such that \( 2\lambda_x \) is the bandwidth of the observer. This observer can be designed using standard Kalman methodology.

V. SYSTEM IDENTIFICATION AND EXPERIMENTS

1) System Identification: The system models are obtained from the frequency response data taken at different solenoid positions. To identify the solenoid model, at each position (measured using a LVDT), we apply biased sinusoidal voltages at various frequencies (from 0.5Hz to 1024Hz) to the solenoids, and measure the corresponding total current output. Sampling times between 0.05ms to 0.1ms are used. Figure 4 shows the frequency response plots at various positions.

According to the eddy current model (4), the position dependence only resides on the LHS via \( L(x) \) which affects the derivative term. In the frequency domain, \( L(x) \) will multiply each \( s \) term. Therefore, by scaling the frequencies by \( 1/L(x) \) in Fig. 5, the frequency response plots should collapse when plotted with this frequency scaling. This is illustrated in Fig. 6. The collapsed frequency response is then fitted with a third order model (4) that incorporates 2 eddy current modes. This shows that the 2nd order model is accurate up to 100Hz (referenced to the middle stroke) whereas the 3rd order model is accurate up to 1024Hz.

A. Testing the self-sensing algorithm

To test the self-sensing algorithm experimentally, open loop tests were first performed. This involves sending input signals to the solenoids and physically perturbing and moving the dual solenoids. A LVDT is used to record the movement while the input signals and physical perturbations are applied. Figure 10 shows some typical results. The input voltage signals consists in a D.C. signal superimposed by high frequency A.C. (Fig. 8). The corresponding current
Fig. 4. Frequency response plots at different positions. Top: Solenoid 1. Bottom: Solenoid 2.

Fig. 5. Inductance values used to collapse the frequency response.

Fig. 6. Inductance scaled frequency responses at 11 locations and the fit with the eddy current models. Top: Solenoid 1. Bottom: Solenoid 2. Solid blue = 3rd order model with 2 eddy current modes. Dashed pink = 2nd order model with 1 eddy current mode.

Fig. 7. Inductances $L_2$ versus $L_1$ at various locations superimposed on the contour lines of $h(L_1, L_2)$. 

Contours of $h(L_1, L_2)$
The RMS error is found to be 0.061mm over the stroke length of 5mm. The next step is to incorporate the self-sensing function and utilize it in closed loop position control.

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