Optimal Operation of a Hybrid Hydraulic Electric Architecture (HHEA) for Off-Road Vehicles Over Discrete Operating Decisions

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Abstract—Most off-highway construction and agriculture equipment use hydraulics, which has unmatched power density, for power transmission and throttling as a means for control. Trends towards better efficiency and electrification motivated a novel Hybrid Hydraulic-Electric Architecture (HHEA) which could significantly reduce energy consumption even in high power machines that would be too costly to electrify directly. This is achieved by using a set of common pressure rails to transmit the majority of power hydraulically and modulating the power with small electric motor-drives to achieve precise control. This paper proposes a computationally efficient, Lagrange multiplier method for computing the optimal sequence of pressure rail selections to minimize energy use. This is needed to evaluate HHEA’s energy saving potential and for iterative architecture design and sizing. An interesting complication is that the cost function is not fully defined until the candidate control sequence is fully specified. This issue is dealt with by decomposing the original problem into a set of subproblems with inequality constraints that can be solved efficiently. A case study of an off-road construction machine demonstrates that the HHEA reduces energy consumption by 2/3 compared to the baseline load sensing architecture.

I. INTRODUCTION

Conventional off-highway mobile machines for the construction, agricultural and turf industries use hydraulic systems for power transmission because of their unmatched power density. Control is often realized using throttling valves which provide precise control but at the expense of efficiency. The average efficiency of existing mobile machines transmissions is only 21% [1] from engine output power. When also considering engine efficiency, machine efficiency will be only 7%.

One approach to improving efficiency is through electrification as the cost of electrical components are becoming more reasonable. This trend is bolstered by some cities limiting the use of internal combustion engines. Benefits of electrification include flexible routing, controllability, energy storage density, and energy regeneration. Despite advances in electric drive technologies, the size and cost of electric machines are still prohibitive for full electrification of the high power mobile machines in which each degree-of-freedom requires 100’s of kW power.

By combining the merits of hydraulic and electric components, a novel architecture was proposed to increase efficiency and maintain control performance. The Hybrid Hydraulic Electric Architecture (HHEA) (Fig. 1) [2] is similar to an Electro-Hydraulic Actuator (EHA) in which an electric motor/generator drives a hydraulic pump/motor to provide

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time. Determining optimal control for drive-train operation is often done using Dynamic Programming (DP) (see e.g. [3] and many others). It is very general and thus can be used for a wide variety of problems. However, it is computationally expensive and thus is infeasible if used within a design optimization. Furthermore, the optimal control is highly dependent on the drive cycle and is thus impractical to apply in real time unless control rules can be extracted from them.

In this paper, the computationally efficient Lagrange multiplier method in [4] is adapted for the present application. This approach requires making simplifying assumptions for the energy storage dynamics. This translates to the accumulator pressures being constant. Besides being orders of magnitude more efficient than DP, the optimal control can be applied in real time. An interesting aspect of determining the optimal operation of the HHEA is that the cost function is conditional on the signs of the net common pressure rail flows which are unknown a priori. In order to cope with this complication in the context of Lagrange multiplier method, the optimization problem is decomposed into a set of constrained sub-problems.

The rest of the paper is organized as follows. Section II describes the HHEA and its mathematical model. The optimal control problem is presented in Section III. The Lagrange multiplier method is derived in section IV. A case study is presented in section V. Concluding remarks are given in section VI.

II. HYBRID HYDRAULIC-ELECTRIC ARCHITECTURE (HHEA)

A. Overview

The idea of the HHEA can be derived by considering an electrohydraulic actuator (EHA) in which the motion of the actuator is dictated by the flow to both sides of the cylinder and the pump/motor must produce the full pressure required by the actuator. This results in high torque on the pump/motor. If the pump/motor could be connected to an elevated pressure, the electric motor driven pump/motor, would only need to produce the difference between the elevated pressure and the required pressure resulting in smaller torque and power requirements for the electrical components. This is the goal of HHEA.

To realize this idea, the HHEA utilizes multiple common pressure rails (CPRs) (Fig. 1 top). An accumulator is connected to each pressurised CPR to help maintain pressure as flow enters/exits a rail via the main pump or the actuators. The main pump can be connected to any of the pressurized rails using valves and is driven by an engine, an electric motor, or a combination of the two. Hydraulic-Electric Control Modules (HECMs) moderate the power from the CPRs to meet the demands of each actuator.

Each HECM consists of an electric motor connected to a fixed-displacement hydraulic pump/motor (Fig. 1-bottom). A set of valves determine which two pressurized rails are used for the linear/rotary actuator to which the electric motor- hydraulic pump/motor combination (eMPC for short) is connected. The eMPC regulates the hydraulic energy to meet the demand of the actuator by either motoring or pumping. The eMPCs need account for the difference between the drive cycle force/torque and the force/torque that the common pressure rails would provide. By choosing pressure rails that result in a force/torque close to the drive cycle requirements, the electric motor of the eMPC can be significantly downsized when compared to a fixed displacement EHA. The HECM should always perform at least as well as an EHA as the HECM can always operate as an EHA would simply by selecting both CPRs as tank.

B. System Model

Referring to Fig. 1 (top and bottom), each degree-of-freedom of the machine is served by a hybrid hydraulic-electric control module (HECM). Each HECM is connected, selectively, to two pressure rails, $P_A$ and $P_B$ chosen from the set of $n_R$ common pressure rails.

$$CPR = \{P_{R1}, P_{R2}, \ldots, P_{Rn_R}\}$$

1) Linear Actuator: For a linear actuator, a two-position directional control valve (DCV) determines which sides of the actuator are connected to the pressure rails $P_A$ or $P_B$ via the electric motor-hydraulic pump/motor combination (eMPC). Let $(\dot{x}(t), F(t))$ be the velocity and external load (including dynamic forces) on the actuator. The pressure drop across the HECM hydraulic pump/motor, $\Delta P_{HECM}$ is given by:

$$DCV = 0: \text{eMPC on cap-side:}$$

$$A_{cap}(P_A + \Delta P_{HECM}) - A_{rod}P_B = F$$

$$DCV = 1: \text{eMPC on rod-side:}$$

$$A_{cap}P_B - A_{rod}(P_A + \Delta P_{HECM}) = F$$

where $A_{cap}$ and $A_{rod}$ are the cap and rod side areas respectively.

Combinations of $P_A, P_B \in CPR$ and produce a set of $n_R^2$ (not necessarily unique) discrete actuator forces:

$$F_{rail}(P_A, P_B, DCV) = \begin{cases} 
A_{cap}P_A - A_{rod}P_B & DCV = 0 \\
A_{cap}P_B - A_{rod}P_A & DCV = 1 
\end{cases}$$

The role of the eMPC is to modulate $F_{rail}$ to meet the required drive-cycle force $F(t)$.

The flows through the HECM pump/motor, and from pressure rails $P_A$ and $P_B$ are:

$$DCV = 0 : Q_{HECM} = Q_A = A_{cap}\dot{x}; Q_B = -A_{rod}\dot{x}$$

$$DCV = 1 : Q_{HECM} = Q_A = -A_{rod}\dot{x}; Q_B = A_{cap}\dot{x}$$

The required electric motor/generator’s speed and torque are:

$$\omega_{eM} = \omega_{PM}(Q_{HECM}, \Delta P_{HECM})$$

$$T_{eM} = T_{PM}(\omega_{eM}, \Delta P_{HECM})$$

where $\omega_{PM}(Q, \Delta P)$, and $T_{PM}(\omega, \Delta P)$ are the hydraulic pump-motor’s speed and torque maps.

2) Rotary Actuator: DoFs with rotary actuators are similarly modeled as with the DoF with linear actuators with these exceptions. 1) There are potentially $n_R(n_R + 1)/2$ distinct hydraulic torque levels $T_{rail}(P_A, P_B)$; 2) The sum of $T_{rails}$ and the electric motor torque $T_{eM}$ is the net torque for actuator the rotary load; and 3) $Q_A = -Q_B$. 

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3) Battery power: Given the torque and speed of the electric motor, the total battery power is the sum of the electrical power input to each HECM:

\[ P_{\text{battery}} = \sum_{k=1}^{n_a} T_{eM,k} \cdot \omega_{eM,k} + \text{Loss}_{eM,k}(T_{eM,k}, \omega_{eM,k}) \quad (8) \]

where \( \text{Loss}_{eM,k} \) is the power loss associated with the electric drive and motor and \( n_a \) denotes the number of actuators. Note that the battery power is not dependent on the state of charge. This assumes that the battery is large enough to always provide, or absorb power.

4) Pressure rail flows: The instantaneous flow supplied by pressure rail is given by:

\[ Q_{R,i} = \sum_{k=1}^{n_a} Q_{A,k} \cdot I(P_A = P_{Ri}) + Q_{B,k} \cdot I(P_B = P_{Ri}) \quad (9) \]

where \( I(\cdot) \) is an indicator function such that \( I(\text{cond}) = 1 \) if the logical condition \( \text{cond} \) is true and, \( I(\text{cond}) = 0 \) otherwise.

C. HHEA Losses

1) HECM Losses: The power losses within each HECM consists in the losses in the hydraulic pump/motor and the electric motor and drive. The total HECM losses are therefore:

\[ \text{Loss}_{HECM} = \sum_{k=1}^{n_a} (T_{eM,k} \omega_{eM,k} - \Delta P_{HECM,k} Q_{HECM,k}) + \text{Loss}_{eM,k} \quad (10) \]

2) Main Pump Losses: The main pump is mounted directly on the engine and for off-road vehicles like excavators, the engine operates at constant speed. Therefore, the main pump speed \( \omega_{MP} \) is also a constant. The main pump is connected to each of the pressure rail in a time-multiplexed manner so that flow apportioned to each rail is proportional to how long it is connected to that rail. With an additional directional control valve (not shown in Fig. 1), the main pump can both provide or absorb flow from the pressure rails.

An accumulator is present on each pressure rail to maintain near constant pressure. A key assumption is that the accumulators are large enough to accommodate any transient flow in the opposite direction from that of the net pressure rail flow. This allows the flow between the main pump and each pressure rail to be unidirectional. For example, the net flow re-circulation and additional losses through the main pump. The accumulator may at times absorb transient return flow. The pressure rail to be unidirectional. For example, the net flow. This allows the flow between the main pump and each accumulator may at times absorb transient return flow. The pressure rail to be unidirectional. For example, the net flow.

The main pump losses over the duration of a drive-cycle for each rail is as follows:

\[ E_{MP,i} := \begin{cases} t_{MP,i}^P V_{Ri} & \text{if } V_{Ri} \geq 0 \\ t_{MP,i}^M V_{Ri} & \text{if } V_{Ri} < 0 \end{cases} \quad (14) \]

The main pump losses are only associated with the net pressure rail flow and not with the instantaneous flow.

III. OPTIMIZATION PROBLEM STATEMENT

System operation consists of the set of valve decisions made for each time point. With \( n_s \), linear, \( n_R \) rotary and \( n_s + n_R \) total degrees of freedom, \( n_R \) number of pressure rails, and each service having a directional control valve, the potential number of decisions at each time is \( N = 2^{n_s} n_R^{2n_s} \). For simplicity, the set of options \( D \) are assumed to be time-invariant. Let \( d(t) \in D \) denote the decision variable at time \( t \).

Note that the decision variable contains all information about the selected pressures, \( P_A \) and \( P_B \) as well as the DCV. In section II the losses were shown to be functions of these variables and the drive cycle for a given time. Thus, the losses can all be written as functions of \( t \) and \( d(t) \).

The optimal operation of the HHEA is defined as the trajectory of decisions that results in the minimal amount of input energy for a given finite duration drive cycle such that the hydraulically driven accumulator velocities and stored energy in the system is equal at the beginning and end of the drive cycle. The purpose of the constraint is to ensure that the system operation is repeatable and thus not dependent on the starting charge of batteries or accumulators.

The energy storage constraint manifests as ensuring the battery charge and CPRs charge start and end at the same values. The CPR is guaranteed to end at the same charge if the main pump provides or absorbs the net flow \( V_{Ri} \) in (13) on each rail. This leaves only the battery constraint to be actively managed.

Thus, the optimal control problem is specified as:

\[ \min_{d(t), \tau \in [0,t_f]} \int_0^{t_f} \text{Loss}_{HECM}(t, d(t))dt + \sum_{i=2}^{n_R} E_{MP,i} \text{subject to } \int_0^{t_f} P_{\text{battery}}(t, d(t))dt\tau = 0. \]

where \( \text{Loss}_{HECM}(t, d(t)) \) is the power loss at the HECM at time \( t \) with decision \( d(t) \) defined via (10), \( E_{total} \) is the total energy loss over the drive cycle, and \( E_{MP,i} \) is the net...
energy loss in the main pump associated with providing or absorbing flow for the i-th pressure rail throughout the drive cycle defined in (14).

An interesting aspect of this problem formulation is that the main pump loss $\sum E_{MP,i}$, defined in (14), depends on the signs of the net flows on the pressurized rails. This is a consequence of our desire for not needing explicitly to account for the behavior of the accumulators or to determine the policy for supplying or absorbing flow for the pressure rails. This means that the cost function cannot be evaluated until a complete candidate control sequence is specified. As will be shown in the next section, this complication will be treated as a sequence of sub-problems with different inequality constraints.

IV. DUALITY / LAGRANGE MULTIPLIER METHOD

As formulated in (15), the only dynamics are associated with the constraint in the net battery energy consumption, and assumptions on the directions of the net pressure rail flows $V_{Ri}(d(\cdot))$ used to specify the main pump losses in (14).

A. Battery equality constraint

Assume for the moment that the signs of net flows of the pressurized rails are known beforehand so that the main pump loss terms $E_{MP,i}$ in (14) can be explicitly defined. The battery constraint in (15) is solved using the Lagrange multiplier method of converting a constrained optimization into an unconstrained one. Consider the modified cost function (Lagrangian):

$$J(\lambda, d(\cdot)) = \int_{0}^{t_f} \left[ Loss_{HECM}(t, d(t)) + \lambda \cdot P_{battery}(t, d(t)) \right] dt$$

where $l_{MP,i} \in \{l_{MP,i}^{P}, l_{MP,i}^{M}\}$ is defined according to the assumed signs of the i-th net rail flow (ignoring tank) and $\lambda \in \mathbb{R}$ is the Lagrange multiplier.

Let $d^*(\cdot)$ be the optimal decision trajectory for the assumed signs for the pressurized pressure rail flows that satisfies the battery equality constraint. For each $\lambda \in \mathbb{R}$, since $J(\lambda, d^*(\cdot)) = E_{total}(d^*(\cdot))$, we have:

$$\min_{d(\cdot)} J(\lambda, d(\cdot)) \leq E_{total}(d^*(\cdot))$$

and

$$\max_{\lambda \in \mathbb{R}} \left[ \min_{d(\cdot)} J(\lambda, d(\cdot)) \right] \leq E_{total}(d^*(\cdot)) \quad (17)$$

Under some regularity conditions, the inequality in (17) is an equality so that the original problem (15) with known signs of the pressure rail flows can be solved via:

$$d^* = \arg\min_{d(\cdot)} J(\lambda^*, d(\cdot))$$

where $\lambda^*$ is obtained from the outer optimization in (17).

Remark: Although the max-min problem in (17) is not guaranteed to solve the original problem, we can check if the solution is indeed optimal. Specifically, since the solution must yield a lower bound for the cost, if the solution satisfies the constraint, then it must be optimal for the original problem.

Two advantages of the Lagrange multiplier method are:

1) it is computationally efficient, because for any $\lambda$, the inner minimization in (17) can be computed in parallel for each time $t$ with no coupling between times, and the outer maximization can be found from a 1-dimensional search such as bisection.

2) the optimal $\lambda^*$ value can be used for real-time use as long as the drive cycle statistics are similar to the one used to obtain $\lambda^*$. This is in contrast to other optimal control policies such as those derived from dynamic programming that are highly specific to the drive cycle.

B. Pressure rail flow inequality constraints

Subsection IV-A above addresses the situation when the signs of the non-tank net pressure rail flows $V_{Ri}$, $i = 2, \ldots, n_R$ are known. To account for the fact that signs of $V_{Ri}$ are normally unknown a priori, the optimal control problem (15) will be solved for each possible combination of signs of the net non-tank rail flows:

$$S = \{(s_2, \ldots, s_{n_R}) | s_i \in \{+1, -1\}\}$$

where $s_i = +1$ implies that $V_{Ri} \geq 0$ and $s_i = -1$ implies that $V_{Ri} \leq 0$. For example, with 3 pressure rails, an optimal control problem is solved for the four combinations of signs of the $3 - 1 = 2$ non-tank rail flows:

$$(s_2, s_3) \in \{(+1, +1), (+1, -1), (-1, +1), (-1, -1)\} \quad (20)$$

In reality, the case $(-1, -1)$ can be ignored for drive cycles that perform net positive work. This can be shown using an energy balance.

The optimal solution for the sign combination with the least cost will be the optimal solution for the original problem.

For each of the sub-problems (with an assumed sign combination), it is necessary to also constrain the optimal solution to observe the assumed sign combination. The $n_R - 1$ inequality constraints are: for $i = 2, \ldots, n_R$:

$$s_i V_{Ri}(d(\cdot)) \geq 0 \quad (21)$$

where $s_2, s_3, \ldots, s_{n_R}$ are the assumed sign combinations. To so do, additional Lagrange multipliers for inequality constraints are used. The cost function in (16) is augmented by:

$$\sum_{i=2}^{n_R} s_i \mu_i V_{Ri}(d(\cdot))$$

where $\mu_i \leq 0, i = 2, \ldots, n_R$ are the non-positive Lagrange multipliers. The augmented cost function can be written as:

$$J_s(\lambda, \mu, d(\cdot)) = \int_{0}^{t_f} \left[ Loss_{HECM}(t, d(t)) + \lambda \cdot P_{battery}(t, d(t)) \right] dt + \lambda \cdot P_{battery}(t, d(t))$$

$$+ \sum_{i=2}^{n_R} (l_{MP,i} + s_i \mu_i) \cdot Q_{Ri}(t, d(t)) \quad (22)$$
Here \( \mu = (\mu_2, \ldots, \mu_n) \) and \( l_{MP,i} = l_{PMP,i}^\circ \) if \( s_i = 1 \), and \( l_{MP,i} = l_{MMP,i} \) if \( s_i = -1 \).

Similar to the case with the equality constraint, suppose that \( d^*(\cdot) \) is the optimal feasible solution (that satisfies the signs of the rail flows), for any \( \mu \) such that all \( \mu_i \leq 0 \),

\[
\min_{d(\cdot)} J_s(\lambda, \mu, d(\cdot)) \leq J_s(\lambda, \mu, d^*(\cdot)) = E_{total}(d^*(\cdot))
\]

Hence,

\[
\max_{\mu \leq 0} \left[ \min_{d(\cdot)} J_s(\lambda, \mu, d(\cdot)) \right] \leq E_{total}(d^*(\cdot)) \quad (23)
\]

Under some regularity condition, this becomes an equality. In particular, if the solution of the max-min problem results in a solution that satisfies both the equality and inequality constraints, the solution is optimal.

Suppose that \( \mu_i \leq 0 \), \( i = 2, \ldots, n_R \) are the solutions to the outer optimization, and \( V_{Ri} \geq 0 \) are the resulting rail flows, then we must have the following complementary slackness condition:

\[
\mu_i V_{Ri}(d^*) = 0. \quad (24)
\]

This is because the outer optimization in (23) must result in \( \mu_i = 0 \) if \( V_{Ri}(d^*) > 0 \) and the alternative being \( V_{Ri}(d^*) = 0 \). Because of this, each sub-problem with an assumed \( s \) can be further decomposed into a number of sub-sub-problems treating each inequality constraint as either active (\( V_{Ri} = 0 \)) or inactive (\( V_{Ri} > 0 \)). The former can be dealt with by treating the inequality constraint as an equality constraint, and since the latter corresponds to \( \mu_i = 0 \), it is solved by ignoring the constraint.

For example, for the sub-problem for a 3-rail system with the assumption \((s_2, s_3) = (+1, -1)\) (net flow is positive for rail 2 and negative for rail 3), the four sub-sub-problems are:

\[
\{(V_{R2} > 0, V_{R3} < 0), \quad (V_{R2} = 0, V_{R3} < 0), \quad (V_{R2} > 0, V_{R3} = 0), \quad (V_{R2} = 0, V_{R3} = 0)\} \quad (25)
\]

In practice, the number of sub-sub-problems that need to be solved can be significantly reduced:

- If the sub-sub-problem with fewer constraints results in a feasible solution, then the sub-sub-problems that are more constrained can be ignored since they cannot result in superior solutions. For example, if \( (V_{R2} > 0, V_{R3} < 0) \) results in a feasible solution, then the other 3 sub-sub-problems in (25) can be ignored.
- Some sub-sub-problems that are physically impossible can be ignored. For example, for drive-cycles that require energy input, the cases \( (V_{R2} = 0, V_{R3} = 0) \) and \( (V_{R2} = 0, V_{R3} < 0) \) are physically impossible for there can be no energy input from the pressure rails.
- Some sub-sub-problems are common to other sub-problems. For example, \( (V_{R2} > 0, V_{R3} = 0) \) is also a sub-sub-problem for \( s = (+1, +1) \).

In summary, the optimal control problem as stated in (15) is solved by considering a number of related optimal control problems with different assumptions on the signs of the net pressure rail flows (and constraint on net battery use) and if the constraints are active or inactive. The solution to the sub-sub-problem that is consistent and has the lowest cost is the solution of the original problem. For the system with 3 pressure rails, the set of sub-sub-problems to be solved are illustrated in Fig. 2.

\[ V_{R2} < 0 \quad V_{R2} > 0 \]
\[ V_{R3} > 0 \quad V_{R3} > 0 \]
\[ V_{R2} > 0 \quad V_{R3} < 0 \]
\[ V_{R2} > 0, V_{R3} = 0 \]
\[ V_{R2} = 0, V_{R3} > 0 \]

Fig. 2. The five sub-sub-problems to be solved for a 3-rail system and a drive cycle that requires energy input.

\[ \begin{array}{c}
V_{R2} = 0, V_{R3} > 0 \\
V_{R2} > 0, V_{R3} = 0
\end{array} \]

Fig. 3. Efficiency map of the manufacturer 107cc/rev axial-piston pump/motor operating at full displacement [5].

V. CASE STUDY

The HHEA modeling and optimization method described in Sections II-IV is now applied to a machine with three linear actuators and one rotary actuator. The eMPC for linear actuators are sized such that without any losses peak flow results in an angular velocity of 5000 rpm. The rotary actuator maintains the same size pump currently in use. The hydraulic pump/motor map in Fig. 3 is applied to pumps of different sizes by scaling the torque and flow losses under the assumption that mechanical and volumetric efficiencies for pumps of varying sizes are constant given the same operating point. The electrical component loss map is obtained by assuming a constant efficiency of 90% for both generating and motoring.

When \( n_R = 2 \), the pressure of the single pressurized CPR is set by the peak pressure required by all of the actuators for the entire drive cycle. There is only 1 physically feasible sub-problem (\( V_{R2} = 0 \)) and the optimal occurs when \( V_{R2} \) is strictly greater than 0.

When \( n_R = 3 \), the highest pressure, \( P_{R3} \) is again set by the peak pressure required and \( P_{R2} = \frac{1}{2} P_{R3} \). The results are
shown in Fig. 4 with the optimal operation found using sub-sub-problem 11. Notice that there are only five independent feasible sub-sub-problems and that they correspond to the problems shown in Fig. 2.

It is interesting to note that the operation that minimizes energy use tends to select the rail forces, \( F_{rail} \), that are close to the drive cycle force, \( F_d \), for all time. This is illustrated for one actuator in Fig. 5 for the optimal control of HHEAs with one and two rails.

With two pressurized rails, the optimized decisions are able to maintain a smaller difference between \( F_{rail} \) and \( F_d \) compared to an HHEA with one pressurized rail due to the increase in the number of discrete rail force options. The required HECM electrical motor torque magnitude is 38 Nm with one pressurized rail and only 10 Nm with two pressurized rails.

To demonstrate the optimal operation on system performance, the input energy required for drive cycles A, B, and C is listed in Tab. I. A baseline load-sensing case is provided for each of the three drive cycles as load sensing is the existing architecture used in many off-road mobile hydraulic systems. The HHEA systems offer significant potential energy savings over the current load-sensing systems with input energy reduced by roughly 2/3 for each of the drive cycles.

Drive cycles A, B and C have 2500 time points and 6561 operating possibilities per time point. For these conditions, each of the sub-sub-problems takes 10 seconds. Thus the optimal control strategy can be found in less than a minute for a drive cycle for systems with 3 CPRs.

VI. CONCLUSION

A static model of a novel hydraulic architecture is presented and a constrained optimization problem is formulated for optimal operation. A method was developed to solve for the optimal operation utilizing an efficient implementation of the Lagrange multiplier method. By using complimentary slackness the problem is divided into sub-problems and conditions are defined to determine which sub-problems are feasible, consistent with the assumed form of the cost function, and optimal.

The study of the HHEA is still in early phases. Potential energy savings for several platforms has been verified through the use of this static model optimization. Current work focuses on validating the static model of the system with a dynamic model as well as implementing a real-time controller for optimal operation. After the results are shown to be consistent with the dynamic model, the static model will be used to drive design decisions of a test stand in which control, and energy savings can be experimentally validated.

ACKNOWLEDGEMENTS

This material is based upon work supported by the Department of Energy, Office of Energy Efficiency and Renewable Energy (EERE) under grant: DE-0008384.

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