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DESIGN OF A VARIABLE RADIUS PISTON PROFILE GENERATING ALGORITHM

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ABSTRACT

One of the main sources of efficiency loss in heat engines is the inability of a sinusoidally displaced piston engines to approximate the ideal heat volumetric cycles the engines require. While attempts have been made to address this issue in the past, recent developments in Stirling engine technology utilizing rolling diaphragm seals on the cylinders has offered an opportunity to greatly increase the correlation between an engines volume-time profile to the ideal profile. By changing the radius of the piston used to drive the rolling diaphragm connection over its length, the piston can effectively be used as a "transfer function" translating the sinusoidal displacement of the crankshaft into a near ideal heat cycle volumetric displacement. This work presents a methodology for determining the ideal shape of such a piston, and a model used to most effectively match a desired input linear displacement profile with output volumetric displacement profile, without compromising the operating conditions required to maintain the diaphragm itself.

NOMENCLATURE

- Cp Contact point. The point where the diaphragm first contacts the piston after leaving contact with the cylinder wall
- Cv Convolution radius, the radius of the fold in the rolling diaphragm.
- R(l) The profile of the radius of the piston with respect to its length

- S(t) The axial displacement function of the input piston with respect to time
- Vo(t) The desired volumetric output displacement function with respect to time
- VI(l) The volume length relationship of the piston being developed
- TP Total perimeter of half the rolling diaphragm length
- Al(t) Axial length displacement of input piston with respect to time

INTRODUCTION

When considering thermodynamics cycles, there always exists an ideal cycle that maximizes performance, and a non-ideal cycle, that represents the real life operation of a machine [1]. The work presented here aims to reduce this gap, by modifying the volume-time profile of sinusoidally displaced piston in a liquid piston engine utilizing a rolling diaphragm seal. While developing the liquid piston Stirling [2] engine, it was decided that a rolling diaphragm seal could be used to effectively transfer mechanical energy from the liquid piston to a mechanical piston. A rolling diaphragm seal is a unique seal often used to transfer energy from sealed pressure chamber to a solid manipulator, and functions by inverting itself and "rolling" back and forth over itself without having any sliding surfaces, as shown in Fig. 1 [3]. After designing the implementation of this seal, it was soon noticed that the unique operation of a rolling diaphragm seal could be manipulated to transform the axial displacement

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profile of a crankshaft into a volume-time displacement profile that more closely matched the ideal displacement profile.

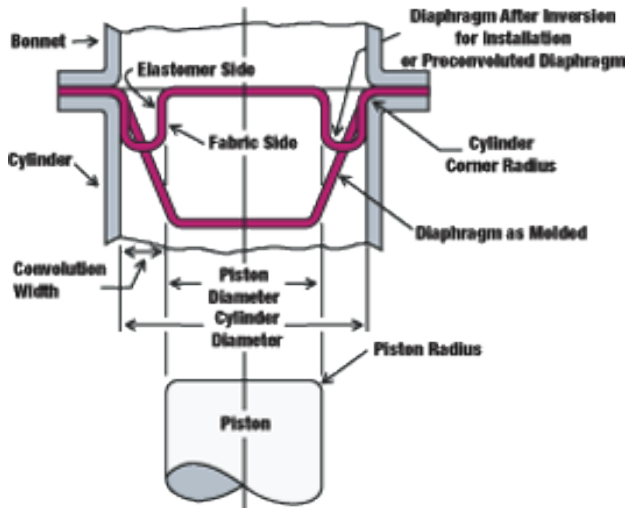


Figure 1. Diagram showing the operation of a typical rolling diaphragm seal, and several of its important variables. (3)

When a straight sided piston is displaced axially, the shape of the volume-time displacement is the same as its axial displacement profile, however if the piston had a changing radius along the axis, the volume-time profile could have a different shape than the axial displacement. With a conventional piston engine, this is not possible because the engine relies on a solid cylinder surface to seal the piston, but with the rolling diaphragm, the radius of the piston can be decreased from its maximum radius to change the piston's cross sectional area with respect to its length. This effectively turns the piston into a transfer function, changing the undesirable axial input profile into a more desirable volume-time output profile. Clearly though, there are limitations on how much modification of the input profile this concept will be able to enact, such as materials properties, length to width ratio of the cylinder, operating pressure and temperature, working fluids and others [3]. The work presented in this paper is the design and construction of an algorithm that will take in information about the application of a specific piston, and generate an optimized piston profile to improve the performance of the piston without exceeding limitations on the materials. While this work has mainly been focused on optimizing performance of liquid piston so it could later be applied to increase efficiency in other processes, such as hydraulic accumulators and air compressors [2].

Model Construction

While this concept mathematically enables large improvements in heat engine efficiency, in reality there are many limiting factors that reduce the effectiveness. The most prominent of these limitations is the material properties of the diaphragm itself, or more specifically the maximum allowable convolution radius. The geometric variables are shown in Fig 1. While the most important factor to track appears to be the convolution radius, another important factor is the derivative of this value. As this derivative increases, so does the stretching of the diaphragm, but also the folding and overlapping of it. The high speed folding and overlapping is the least studied aspect of this design, and will require analysis in future experimental work. A derivation of these equations into their useful form of the model is presented below:

While convolution radius, and other material limitations are all constraints, they do not produce any of the piston geometry, they merely limit it. The product of the work presented in this paper, is a mathematical model that will allow the user to input an axial displacement profile, a volume-time displacement profile, other geometric, material, and system properties of the rolling diaphragm seal, and have an output of an optimized piston profile. The model will also produce an analysis of the adherence to an ideal output, and the identification of areas limited by physical constraints.

Ideal Profile Construction

If it is considered that the ideal rolling diaphragm can stretch around its axis of travel, but not along it, then the perimeter of a cross section of the diaphragm taken at the central axis will remain constant, such that the constant perimeter can derive C_p as shown below:

$$TP = C_p + C_v + \int_{0+}^l \sqrt{1 + \left(\frac{dRl}{dl}\right)^2} dl \Rightarrow \quad (1)$$

$$C_p = TP - \left(C_v + \int_{0+}^l \sqrt{1 + \left(\frac{dRl}{dl}\right)^2} dl\right) \quad (2)$$

The change in displaced volume caused by the change in the convolution radius is estimated as being zero, and the gap between the piston and the cylinder is spanned by a straight line. At this point, nearly none of these terms are defined, so by defining the volume of the displaced piston above the contact point, based on its radius profile the values of the terms may be derived. This is determined as a simple revolved solid

equation. Additionally, the normal angle at the contact point is determined by the derivative of the radius profile. Once these two parameters have been resolved in terms of the radius-length profile, the radius-length profile becomes the only unknown:

$$Vl(l) = \pi \int_0^{Cp} Rl(l)^2 dl \quad (3)$$

The only term on the left side of this equation, is the volume-length profile, but this has not yet been calculated either. In order to define this function, we need to combine the volume-time profile of the ideal cycle, with the axial displacement-time profile of the mechanism to produce a volume-axial displacement profile. In order to accomplish this, we simply invert the axial displacement function so that it is in terms of time-axial displacement. Then we substitute the transformed equation into the volume time profile:

$$Al(t) = l \Rightarrow Al^{-1}(l) = t \quad (4)$$

$$Vl(l) = Vo(Al^{-1}(l)) \quad (5)$$

The resulting equation shown above represents the appropriate amount of volume that should be displaced based on a given input linear displacement. To obtain the relationship between piston radius and length, we simply set the two previously derived equations equal to each other, and solve for R(l) as shown.

$$Vo(Al^{-1}(l)) = \pi \int_0^{Cp} Rl(l)^2 dl = Vl(l) \quad (6)$$

This yields an equation that can generally be solved depending on the complexity of the inputs. To obtain the ideal radius profile, we simply solve for R(l) as shown.

$$\sqrt{\frac{d(Vo(Al^{-1}(l)))}{\pi dl}} = Rl(l) \quad (7)$$

At this point the ideal piston radius profile has been defined, and it is time to begin transforming it into a profile that satisfies the limiting criteria developed earlier.

Ideal Profile Adjustment

The profile adjustment phase of this work, is when the limiting constraints developed earlier come into consideration. The first step is to determine the outer cylinder radius in terms of the minimum rolling diameter of the diaphragm and maximum radius of the piston. After this is calculated, the rolling radius vs. length plot is developed, and the maximum allowable value is overlaid. The maximum values for the convolution radius (CvMAX) based on the diaphragm tensile strength (DTS) is defined below.

$$CvMAX = \frac{DTS}{2 * WorkingPressure} \quad (8)$$

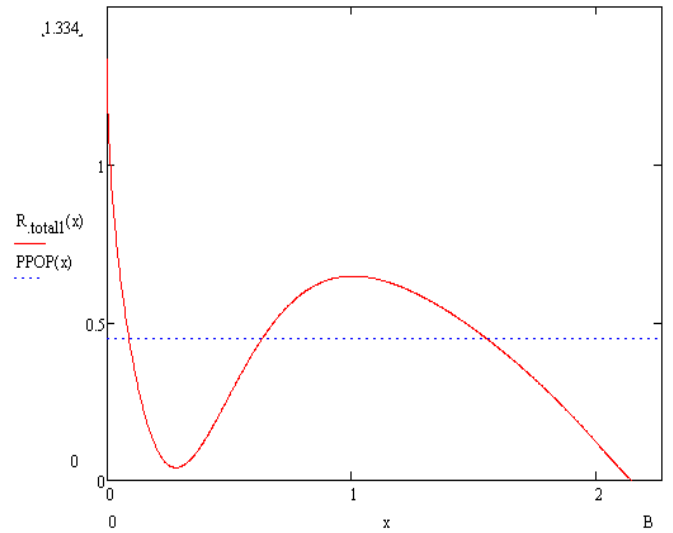


Figure 2. Plot of the convolution radius from an ideal piston, with the max radius of curvature overlaid.

Wherever the rolling radius plot is above the maximum allowable rolling radius as shown in Fig. 2, the diaphragm is spanning too large of a gap unsupported, and presents a possibility of failure. The exception to this is in the beginning of the plot, where the high values of the convolution radius represent the radius converging to zero indicating the end of the piston. Oversized rolling radius areas can be reduced by truncating the maximum piston radius and decreasing the cylinder radius. In Stirling cycle engines, this corresponds to the bottom end of the piston which effectively "rounds off" the sharp point at the bottom of the displacement curve. A representation of trimming the maximum height off the end of the piston, and the effect it has on the volume time, and pressure volume plot is shown in Fig 3

The area rounded off from this plot is a direct contributor to loss of work output [1] and should be limited if possible. The

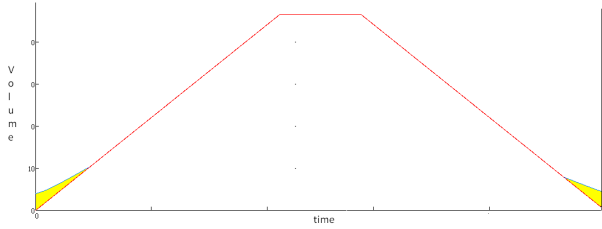


Figure 3. A representation of the effects of reducing the high tip of the piston to decrease average convolution radius.

next type of problem that can be encountered is an oversized rate of radius change. This generally needs to be "smoothed out" by segmenting the profile into three sections, the problem area, and the areas in front of and behind the problem area. The problem area is defined by finding the point of the most curvature, then expanding outwards in equal arc lengths along the curve. The curve is traced until the maximum slope of a polynomial curve between the two endpoints is within acceptable limits. The endpoint values on the curve, and the corresponding slope of the radius profile is used as the boundary conditions to develop this polynomial equation. Other types of equation could be used to span the removed sections of the curve, but have not yet been investigated. Shown in Fig 4 is the area being reshaped, followed by the relevant calculations:

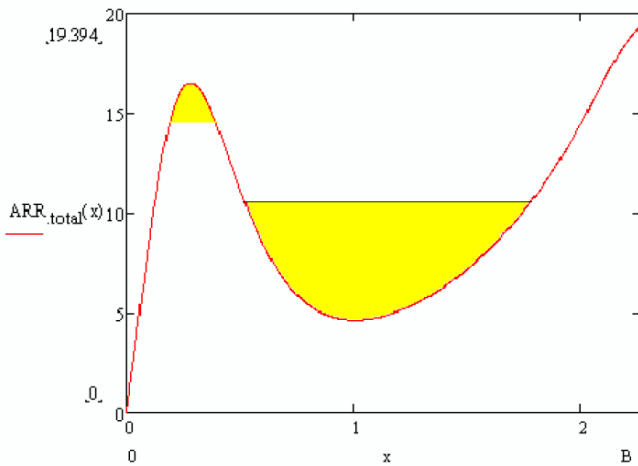


Figure 4. A possible example of a high slope area in a plot, with the values of ARR representing the radius height, and Acor representing the slope at those values.

$$ARR(y) = A^2x + Bx + c \quad (9)$$

From these boundary conditions, we can easily solve for a relation for the parameters A,B, and C that allows for the adjustment of the shape of the curve manually.

After these problem sections are resolved the final radius is

ready to be converted into a solid model to produce a part. Currently, the algorithm is set up to produce a large series of datapoints defining the radius profile of the piston, which are then imported to a solid modeling program and revolved into a solid model.

Results

In order to best display the results of this work, its functionality will be demonstrated, by generating a variable radius piston for the liquid piston Stirling engine. The first step as noted above is to input parameters.

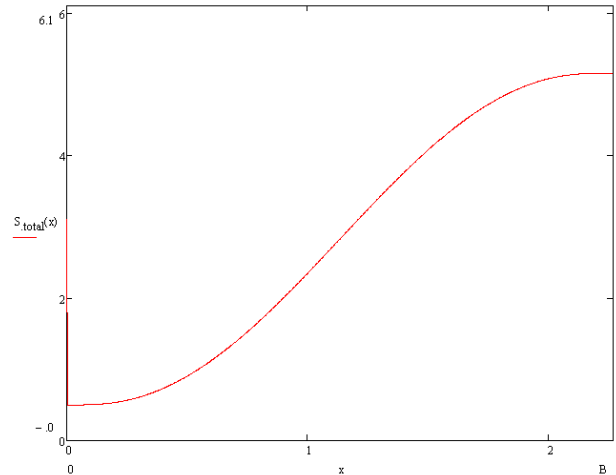


Figure 5. The axial displacement vs. time of the input piston.

It should be noted that a cam with a modified sine profile has been employed to displace the piston. Due to this, the rise and fall of the piston are of symmetrical, so only the rise is modelled. Since the dwell in the ideal cycle is held by the cam profile, only the dynamic area of the ideal volumetric plot is considered, which shown in Fig 6. The additional device parameters have been entered into the model, but will not be discussed specifically. The next step is to plot the ideal piston radius profile, shown in Fig. 7 The next step is to identify problem areas. Note that there is a large problem area indicating too large of a Cv. This entire area is reduced by reducing the height of the trailing edge peak, as discussed above. The small area remaining was close to needing reshaping through the polynomial method mentioned above, but this was not necessary.

At this point, this is the first draft of the final piston profile. Fig. 9 demonstrates the ideal displacement profile, the proposed pistons displacement profile, and the straight sided piston profile.

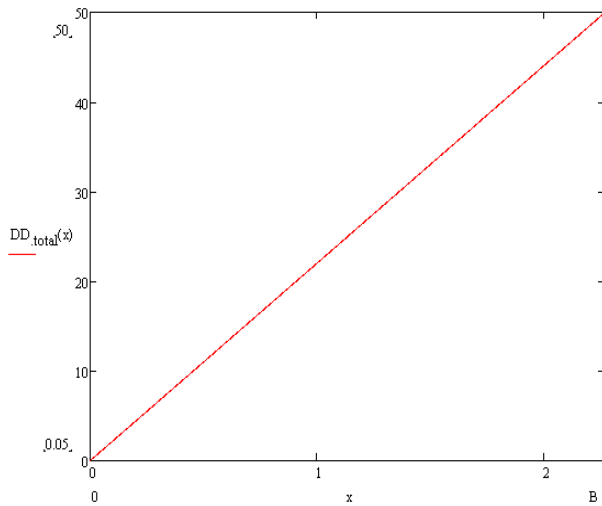


Figure 6. The desired shape of the volume rise in the proposed engine.

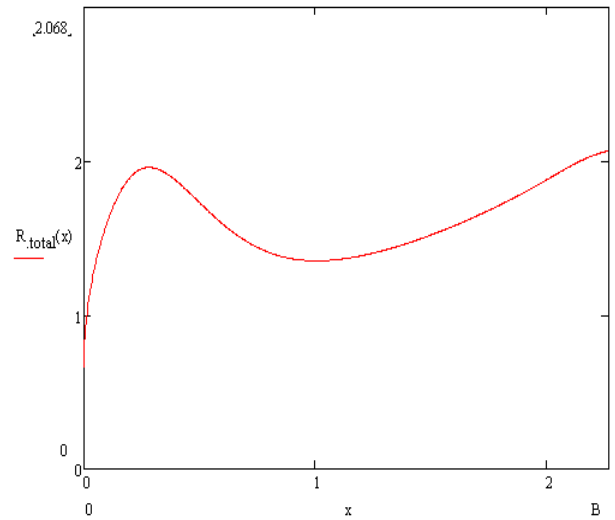


Figure 8. The final piston radius profile to be cut for testing, after adjusting maximum height offsets.

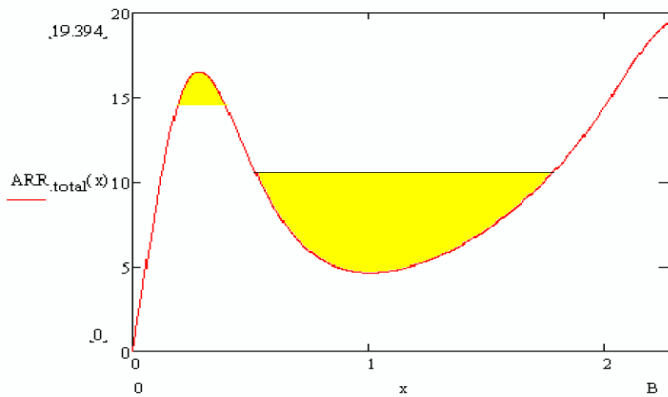


Figure 7. The ideal piston profile without considering materials properties.

As can be seen the new piston is not completely ideal, but the deviation from the ideal profile is significantly reduced.

1 Conclusions and Future Work

This cycle optimization process appears to show great promise in decreasing Stirling engine deviation from ideal profiles, though it is important to note that there are several considerations that have not yet been addressed in this paper. The first and foremost is that this model assumes a constant volume to the convolution radius. While the changes in the displaced volume caused by a changing convolution radius is small, it could still be significant,

and currently the model is being modified to take this into account. The second issue is that the limiting factors being implemented by the model currently do not take into account the dynamic operation of an engine and are simply tuned for the engine's maximum potential values. Greater rolling radii can be achieved when the cylinder is at lower pressure, and a

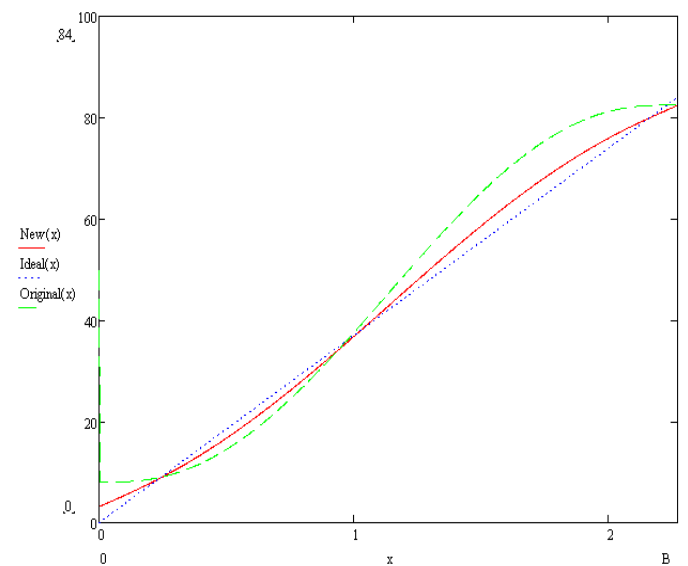


Figure 9. The volume-time displacement of the new piston, compared to the ideal cycle shape, and the original profile.

dynamic rolling radius limit needs to be developed to reflect the changing pressure inside the engine. Additionally, there has been no analysis of impact moments of the fluid when the liquid piston changes direction. There is a potential for high momentary forces at the diaphragm when this happens that will be based on the operational frequency of the engine. Finally, while this piston has been mathematically optimized, it has not yet been tested, which leads me to the future work.

The next step in this work is to finish building the test equipment and integrating the algorithm modules so that we can begin testing the voracity of our theories. After these first tests are completed and we have a better understanding of the mechanisms of the design, our next round of optimized testing will be geared towards several kinds of applications to compare

effectiveness such as a Stirling engine, an air compressor, and a hydraulic accumulator.

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