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SYNTHESIS OF A VARIABLE DISPLACEMENT LINKAGE FOR A HYDRAULIC TRANSFORMER

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ABSTRACT

A hydraulic pump/motor with high efficiency at low displacements is required for a compressed air energy storage system that utilizes a liquid piston for near-isothermal compression. To meet this requirement, a variable displacement six-bar crank-rocker-slider mechanism, which goes to zero displacement with a constant top dead center position, has been designed. The synthesis technique presented in the paper develops the range of motion for the base four-bar crank-rocker, creates a method of synthesizing the output slider dyad, and analyzes the mechanisms performance in terms of transmission angles, slider stroke, mechanism footprint, and timing ratio. It is shown that slider transmission angles can be kept above 60 degrees and the base four-bar transmission angles can be controlled in order to improve overall efficiency. This synthesis procedure constructs a crank-rocker-slider mechanism for a variable displacement pump/motor that can be efficient throughout all displacements.

INTRODUCTION

Utilizing compressed air energy storage (CAES) for renewable energy generation, such as wind turbines, eases the variation between power demand and wind availability. This is done by storing energy during periods of high production and low consumption and releasing this stored energy during periods of low production and high consumption [1]. CAES requires the compression of atmospheric pressure gas to high pressure and then the expansion of gas from high pressure back to atmospheric pressure. Extreme gas temperature variations

are created when the compression or expansion is done adiabatically. When operating at a working pressure of 35MPa the final temperature after compression and expansion are found to be 1593K and 56K respectively. This is too large of a temperature variation for normal materials to handle for multiple cycles, driving costs for this system with specialty materials engineered for the extreme temperature cycling. Thermal storage methods would be necessary in the system to recirculate the associated thermal energy in order to maintain the overall system efficiency.

The proposed system uses a liquid piston and/or a liquid spray to maintain isothermal compression and expansion for efficiency. The liquid piston and spray would act as a thermal sink to capture the heat generated during compression, and a thermal source to provide heat during expansion. By using a large quantity of small droplets, high surface area for heat transfer is provided. After impinging on the compressor, these droplets fall into the liquid piston allowing for the removal of this heat in the closed system [1].

To control the liquid piston, a “hydraulic transformer” has been proposed by Sancken and Li. The transformer acts as a pump/motor to pump fluid to the liquid piston during compression, and convert fluid pressure into mechanical power during expansion. Previous work proposed a trajectory of compression, optimized for efficiency, relating volume/volume max to total cycle time [2]. As shown in Figure 1, the displacement curve features a rapid volume change for the beginning of the cycle followed by a more gradual volume change near the end of the stroke.

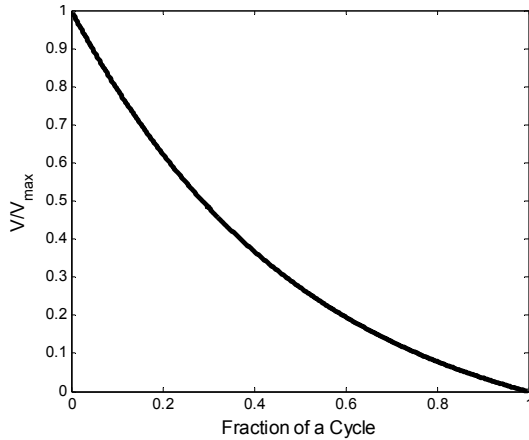


FIGURE 1. DISPLACEMENT TRAJECTORY FOR OPTIMAL COMPRESSION [3]

As a result of the desired displacement trajectory, the pump/motor displacement is between 0 and 20 percent of maximum displacement for ~45 percent of the cycle. As described by McCandlish, the mechanical efficiency of a hydraulic pump/motor is described by:

$$\eta_{mech} = \frac{1}{1 + \left(\frac{c_v \times v \omega}{x} \right) + \frac{c_f}{x}} \quad (1)$$

where c_v and c_f are the viscous and dry coefficients of drag, v is viscosity, ω is angular velocity, P is pressure, and x is the fraction of displacement [4]. For existing variable pump/motor architectures, the maximum mechanical efficiency is only achieved during a small fraction of the compression cycle. A table of the mechanical efficiency for a typical axial piston pump through a three stage compression cycle is shown in Table 1. It shows efficiency can reach as low as 36% with an average efficiency of 69%. To be successful for this application, a pump/motor with high efficiency at low displacement is required.

TABLE 1. EFFICIENCY OF A TYPICAL AXIAL PISTON PUMP/MOTOR THROUGH THREE STAGES OF COMPRESSION

Stage	Efficiency Range	Average
1	36% to 39%	38%
2	72%-80%	77%
3	83%-95%	90%
Overall Average: 69%		
$c_v = 4.9e5$	$c_f = 0.024$	$v = 1e-3 \frac{N*s}{m^2}$

There are three main architectures currently available for variable displacement pumps. The axial piston pump uses a swashplate at various angles which acts as a cam to change the travel of pistons which displace fluid. As the angle of the

swashplate increases, pistons displacement also increases. The bent axis piston pump uses a cylinder block which is off axis from the drive shaft. The pistons are mounted to a disk which is in line with the drive shaft and all components spin causing the pistons to reciprocate due to the bent axis. The angle of this axis determines the displacement. The vane pump uses a cam ring and an internal, eccentric cylinder which are separated by vanes which create fluid chambers. As the eccentricity is increased, the fluid chambers become larger and more fluid is displaced. All of these pumps have poor efficiencies at low displacements.

An alternative approach to existing variable pump/motor architectures is to create an adjustable crank slider linkage which can vary its stroke and thus the displacement. Synthesis techniques of adjustable mechanisms have been addressed previously. Tao and Krishnamoorthy developed graphical synthesis technique for generating adjustable mechanisms with variable coupler curves [3; 5]. McGovern and Sandor presented a complex number method to analytically synthesize adjustable mechanisms for variable function and path generation [6; 7]. Handra-Luca outlined a design procedure for six-bar mechanisms with adjustable oscillation angles[8]. Zhou and Ting present a method of generating adjustable slider-crank mechanisms for multiple paths by adjusting the distance between the slider axis and the crank [9].

Similar work has been done, and several patents have been awarded, for internal combustion engines which vary the displacement of a piston while maintaining a constant compression ratio depending on the power demand [10-13]. These engine linkages, however, do not go to zero displacement. Shoup developed a technique for the design of an adjustable spatial slider crank mechanism for use in pumps or compressors[14]. This mechanism though requires the repositioning of the axis of slide relative to the crank. None of these mentioned techniques and examples simultaneously allows for constant top dead center and zero displacement.

This paper describes the synthesis of a variable displacement slider linkage which can drive a piston with a constant top dead center (TDC) and achieve zero displacement. In the first section, the principles of such a linkage are presented, which drives the type synthesis. The second section describes the dimensional synthesis of the linkage. The third section describes the task specifications and how they drive synthesis. The fourth section will discuss the performance results of several linkages. The fifth section contains concluding remarks.

METHODS

Type Synthesis

The goal of this linkage is to vary one of the links such that there is a change in the displacement or stroke of a slider (the piston). In order to guide the synthesis process, several

requirements must be defined. For simplicity, a ground pivot, as opposed to a moving link, will be used to vary the displacement of the linkage. The axis of slide of the piston and the ground pivot of the crank will remain fixed to simplify the pump/motor block. Therefore, a third, movable, ground pivot is required for a functioning variable linkage. A minimal number of links is favorable as it limits the linkages complexity, moving-mass, and overall size. The linkage options are thus limited to either a Watt II or Stephenson III six-bar with a slider output, as both of these linkages have three ground pivots. Of these 2 linkages, the Stephenson III seems more favorable because the ground pivot available for repositioning is attached to a binary link rather than a ternary resulting in less moving mass to vary when the linkage is in motion.

Task Specification

With a linkage type selected, the next focus is on the task specifications, the first of which is to achieve zero displacement. In order to create a slider mechanism with zero displacement, it is necessary for the coupler point to move in an arc about the pivot of the slider. As a result, the coupler point will not impart any translation on the slider. If a fourbar, crank-rocker mechanism is constructed such that the coupler curve is a pseudo-arc, and the slider pivot is placed at the center of this arc, then the slider of the resultant six-bar linkage would have zero displacement. If the ground pivot of the rocker link of the base fourbar is then moved, the coupler curve would deviate from an arc and therefore cause motion in the slider.

An effort was made to create a fourbar linkage with a pseudo-arc path using 3 and 4 precision position analytical path synthesis, but none of the results produced a true zero displacement linkage. From this effort it was realized that the joint C at the end of the rocker travels in a perfect arc about joint D. Thus, if the pin of the slider is located at point D, and connected to point C with a link, the slider of the resultant mechanism exhibits zero displacement. In this configuration, the rocker link will be overlapping the connecting rod to the slider. By placing the precision position K at joint C, the coupler link as also reduced in mass as it is now a binary link rather than a ternary. Such a linkage can be seen in Figure 2. This is actually a Watt II rather than the prescribed Stephenson III but the moving mass issue is addressed because the ternary link is infinitely small.

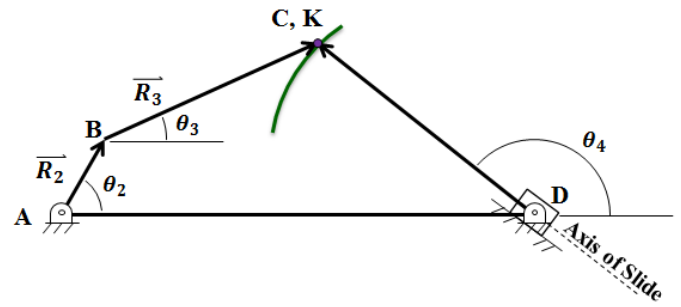


FIGURE 2. ZERO DISPLACEMENT LINKAGE

The second task specification of focus is ensuring the piston has the same top dead center (TDC) position regardless of displacement. A more general case can be used to describe how this is accomplished. In any fourbar crank rocker, such as that in Figure 2, for any position of the input link, θ_2 , the precision position K will be at a defined position with a resultant θ_3 and θ_4 . If the ground pivot of link 4 is rotated about this point K such that there is an arc of D positions, then the coupler point will always return to this same position at the associated θ_2 . Since D is a ground pivot, \vec{R}_1 , a vector describing the ground link, is a function the position of D as it is the vector connecting the ground pivots A and D.

Dimensional Synthesis

To represent this linkage mathematically, the input link is set to unity, and all other lengths are a multiple of this value. The input crank ground position is defined to be the origin of a global coordinate system. The linkage is constructed in five steps:

1. The input fourbar crank-rocker is synthesized
2. The timing of TDC is defined
3. The variable ground pivot locations are found
4. The location of zero displacement and axis of slide are defined
5. A resultant rocker-slider dyad is constructed

Complex number vector notation, using appropriate rotation and stretch operators, is used throughout the synthesis.

Variable Fourbar Synthesis. The mechanism must be a crank-rocker and therefore meet the Grashof condition which is defined by the equation [15]:

$$S + L \leq P + Q \quad (2)$$

where S is the length of the smallest link, L is the length of the longest link, and P and Q are the lengths of the remaining links. Another requirement of a crank-rocker is that link 2 is the shortest link [15]. With link 2 representing unity, and link lengths 3 and 4 chosen, the Grashof requirement defines the

maximum and minimum lengths of the ground link, link 1 where:

$$R_{1max} = P + Q - S \quad (3)$$

$$R_{1min} = S + L - P \quad (4)$$

Thus the fourbar is defined with the range of acceptable values for the length of link 1 being R_{1min} to R_{1max} . The point K will always travel in a circular arc with radius $|\overline{R_4}|$ about ground pivot D.

Defining Top Dead Center. To create an arc of positions of D that can be associated with TDC, K must be located at either end of its associated coupler curve. K is located at one of these ends when $\overline{R_2}$ and $\overline{R_3}$ are colinear in either the extended case or the overlapped case as seen in Figure 3. This is one of the free choices of the linkage.

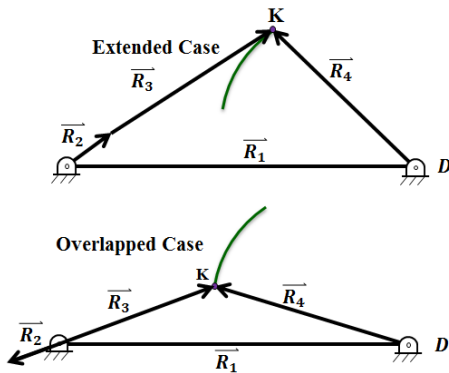


FIGURE 3. EXTENDED AND OVERLAPPED EXTREMES OF FOURBAR LINKAGE

The collinear condition is required because the ends of the coupler curve represent the extreme positions of the rocker, which ultimately control the slider position of TDC and bottom dead center (BDC). If a circle is then created, centered at point K in either of the above cases, with a radius of $|\overline{R_4}|$, and D is at any point along this circle, K will be in the same position at the associated crank angle θ_2 , which is measured relative to the x-axis.

Find the Variable Ground Pivot Locations.

With R_{1min} , R_{1max} , and the timing of TDC defined, the arc of acceptable ground pivots can be defined by determining where the circle of D positions and the circles of radius R_{1min} and R_{1max} , centered at the origin, cross. The arc bounded by the R_{1min} and R_{1max} circles represents the arc of acceptable ground pivots. This will result in two arcs; either of which can

be used as they result in the same linkage but in a different orientation as seen in Figure 4.

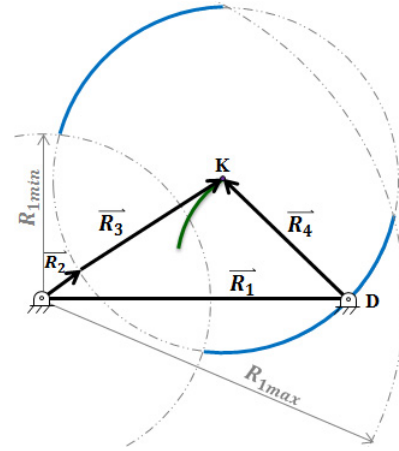


FIGURE 4. VARIABLE GROUND PIVOT LOCATIONS
Define Location of Zero Displacement and Axis of Slide. $|\overline{R_1}|$ Should be set to either R_{1min} or R_{1max} when determining the location of zero displacement and the axis of slide. Doing so means that the position of D associated with zero displacement is located at an extreme of $\overline{R_1}$. This allows maximum travel of D as it moves to the opposite extreme of $\overline{R_1}$. Also, the axis of slide can be set at any angle. However, this affects the slider displacement and transmission angles.

Rocker Slider Dyad Construction. Because the axis of slide can be found with the linkage in the extended case or the overlapped case and at R_{1max} or R_{1min} , there are four configurations in which the linkage can be constructed which are shown in Figure 5.

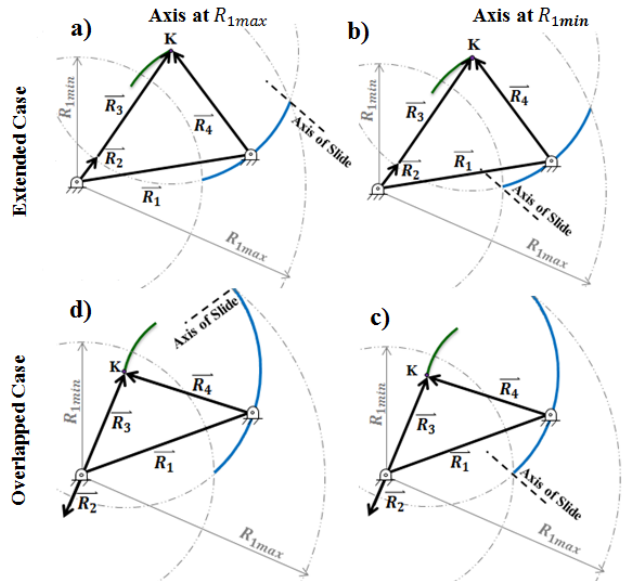


FIGURE 5. CONFIGURATIONS OF VARIABLE LINKAGE

Constructing the slider dyad is done by adding a link equal in length to link 4 called the connecting rod. One end of the connecting rod is pinned to the base fourbar at point K, and the other end is pinned to a slider which travels along the axis of slide. When D is coincident with the axis of slide, no translation is imparted to the slider because K travels in an arc about the slider. As D moves away from the axis of slide, the path of K varies. As a result, the slider is translated along the axis. The slider will always return to TDC at a specific value θ_2 .

Linkage Analysis

For the rest of this paper the linkage will be defined in the configuration with the axis of slide associated with R_{1max} and the extended case as shown in Figure 5a. For ease of calculation and graphical representation, \vec{R}_{1max} is set to be collinear with the x axis and then \vec{R}_1 strays off this axis as it travels along the arc of acceptable ground pivots. To find the position of K associated with TDC, a triangle is created, seen in Figure 6, where $|\vec{R}_{1max}|$, $|\vec{R}_2| + |\vec{R}_3|$, and $|\vec{R}_4|$ are used to find θ_k using the law of cosines.

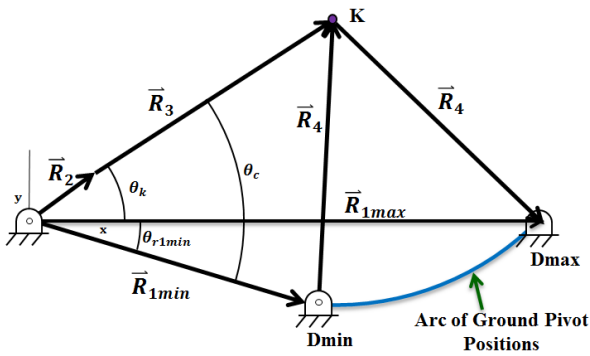


FIGURE 6 ASSOCIATED TRIANGLE FOR DETERMINING θ_k AND θ_{r1min}

The position of K would then be defined as:

$$\vec{K} = (|\vec{R}_2| + |\vec{R}_3|)e^{i*\theta_k} \quad (5)$$

With \vec{K} defined, the arc of acceptable ground pivots can be created. The angle of \vec{R}_{1max} is defined as 0° from the x axis. The angle of \vec{R}_{1min} is determined by making another associated triangle, seen in Figure 6, with sides of R_{1min} , $|\vec{R}_4|$, and $|\vec{R}_2| + |\vec{R}_3|$.

The law of cosines can then be used to find θ_c . The angle of \vec{R}_{1min} from the x axis is then:

$$\theta_{r1min} = \theta_k - \theta_c \quad (6)$$

Therefore:

$$\vec{R}_{1min} = R_{1min}e^{i\theta_{r1min}} \quad (7)$$

Any point between \vec{R}_{1max} and \vec{R}_{1min} at distance of $|\vec{R}_4|$ from \vec{K} would be an acceptable ground pivot position of the rocker of the fourbar linkage. These position vectors can be found for any value of \vec{R}_1 by repeating the previous calculation for θ_{r1min} and \vec{R}_{1min} but replacing R_{1min} with any value in the range of R_{1min} to R_{1max} . Standard position analysis can then be used to determine the link positions and slider displacement at any crank angle and position of ground pivot D. This analysis was done in MATLAB and an image of one resultant linkage can be seen in Figure 7.

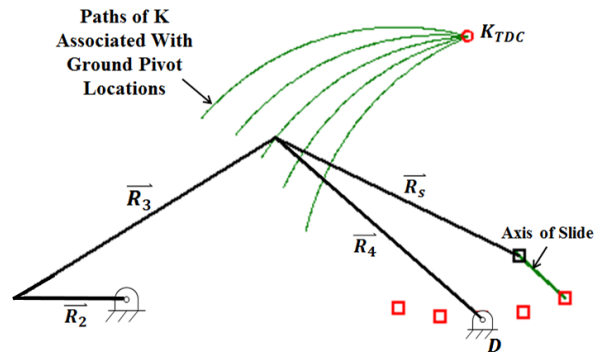


FIGURE 7. MATLAB LINKAGE CONSTRUCTION

Design Parameters

Any value for the lengths of links 3 and 4 can be selected. Specific parameters were chosen to compare one linkage to another. These properties include: minimum transmission angle of the fourbar and slider, maximum displacement of the slider, area of mechanism footprint, and timing ratio. A “better” linkage would have maximized transmission angles, maximized displacement, minimum mechanism footprint, and a timing ratio closest to one.

Transmission Angles. A transmission angle is defined as: *The acute angle between the output link and the coupler link.* [16] Force is best transmitted through these links when this angle is greatest, with the maximum being 90° . There are two transmission angles of interest in this linkage, the angle between links 3 and 4 and the slider transmission angle. Both of the transmission angles can be calculated using standard position analysis techniques. For the base fourbar, the minimum transmission angle occurs when the links 2 and 3 are collinear in either the extended or overlapped case as described previously.

The slider transmission angle is dependent on the angle of the axis of slide. When the connecting rod is in-line with the axis of slide, the transmission angle is optimal at 90°. In the zero displacement configuration of the linkage, the connecting rod travels in an arc about the slider, defined as θ_{4s} . If the axis of slide is placed at an angle, θ_{As} , such that it is pointed at the center of this arc, as seen in Figure 8, the angle between the connecting rod and the axis of slide can be minimized and thus maximize the transmission angles of the slider.

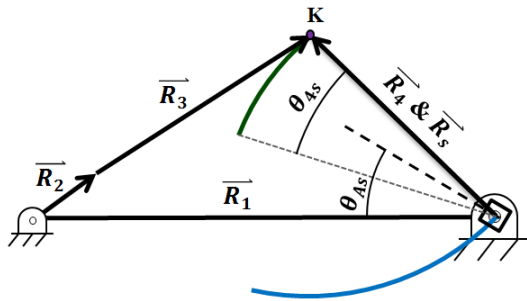


FIGURE 8. DEFINING ANGLE OF AXIS OF SLIDE

θ_{As} can be found using the equation:

$$\theta_{As} = \pi - \theta_{4max} - \frac{\theta_{4max} - \theta_{4min}}{2} \quad (8)$$

Where θ_{4max} occurs at the overlapped case and θ_{4min} occurs at the extended case of the linkage. As the location of D changes, the minimum transmission angle remains constant and occurs at the extended case of the linkage.

Displacement. The slider displacement is defined as the distance traveled by the slider along the axis of slide from the TDC position. The maximum displacement of the slider is calculated at the overlapped case when D is located at the furthest position from the axis of slide. A large displacement is preferred. For comparison, in a standard crank-slider, the displacement of the slider is equal to twice the crank length when the axis of slide passes through the origin.

Area of Linkage Footprint. The footprint of the linkage is defined as the two-dimensional area occupied by the linkage throughout the range of motion and includes the entirety of the linkage. This area is found using MATLAB by plotting the extents of the linkage and then using the *polyarea* function. The units of the footprint are unit length squared.

Timing Ratio. The timing ratio is defined as the ratio of time of working stroke to time of return stroke [15] as seen in Figure 9. When the timing ratio is greater than one, the working stroke is longer than the return stroke which is desirable because it is related to an increased mechanical advantage. When the timing ratio is less than one, the working stroke is

shorter than the return stroke resulting in a lower mechanical advantage. When the timing ratio is equal to one, the working stroke is equal to the return stroke and the linkage timing is balanced. Because this mechanism is used for both compression and expansion, a timing ratio near one is desired.

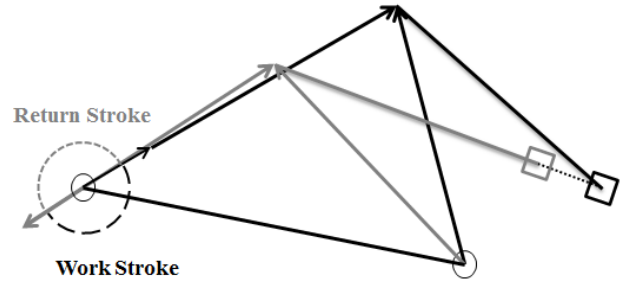


FIGURE 9. LINKAGE TIMING RATIO

RESULTS

The maximum displacement, footprint area, minimum slider transmission angle, and minimum timing ratio were calculated with the lengths of links 3 and 4 set to various units of length. The results are presented in Table 2.

TABLE 2. RESULTS OF LINKAGE

		Ex 1	Ex 2	Ex 3
Parameter	Units	$R_3 = 2$ $R_4 = 3$	$R_3 = 4$ $R_4 = 5$	$R_3 = 7$ $R_4 = 8$
Minimum Slider Transmission Angle	Degrees	66.06	71.56	75.52
Minimum Fourbar Transmission Angle	Degrees	0	0	0
Maximum Displacement	Unit-Length	2.12	5.53	10.19
Footprint Area	Unit-Length ²	10.95	37.97	110.59
Minimum Timing Ratio	Unit-less	.2566	.2781	.2988

Figure 10 shows the maximum and minimum transmission angle of the base fourbar linkage from example 1 versus the length R_1 . The minimum transmission angle is the metric of interest as it shows the worst case scenario of the transmission angle. The minimum is shown to begin at 0° and increase until ~55° and then decrease back to 0°.

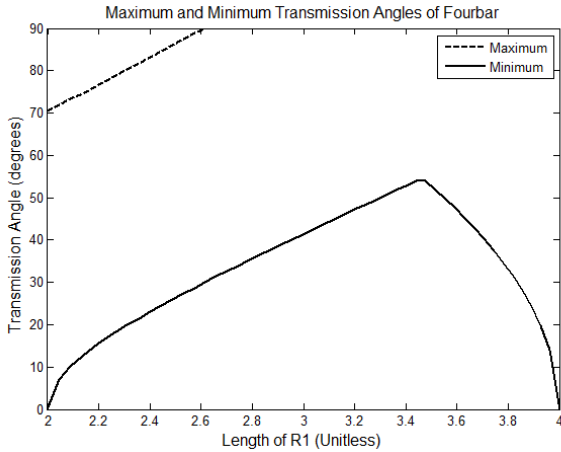


FIGURE 10. MAXIMUM AND MINIMUM TRANSMISSION ANGLES OF BASE FOURBAR

The timing ratio of example 1 is plotted against link length R_1 in Figure 11.

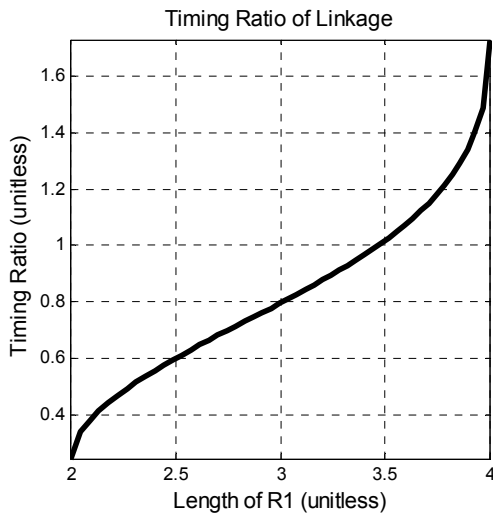


FIGURE 11. TIMING RATIO OF LINKAGE

The plot shows the minimum of .26 and then the maximum of about 1.73 as the plot moves from R_{1min} to R_{1max} which were calculated to be two and four respectively.

DISCUSSION

As a result of using the Grashof condition to define R_{1max} and R_{1min} , the transmission angle will always go to zero, starting and ending in a toggle position at these extreme values for the length of \bar{R}_1 . As D moves away from these extremes though, the minimum transmission angle increases until it

reaches a maximum and then goes back to zero as it moves towards the other extreme. This means that it is possible to define a new R_{1min} and R_{1max} based on the minimum desired transmission angle. This will make the arc of acceptable ground pivots smaller, thus limiting the variability of \bar{R}_1 and therefore the maximum stroke of the mechanism. Reducing the range of R_1 also shrinks the footprint of the linkage. The transmission angle of the slider has been maximized at the position of zero displacement which is beneficial because, at higher displacement levels, the slider will have better transmission angles.

The displacements of the example solutions are larger than two crank lengths which, is greater than a standard crank-slider. This is a promising result, but it must be noted that correcting the fourbar transmission angles decreases this value.

The footprint area of example 1 is only 10.95 square units of length. The area of a comparable crank-slider, with the same maximum displacement and length of link 3, occupies 5.14 square units of length. This means that the variable displacement linkage is twice the size of a fixed displacement linkage. The other examples show much larger footprint areas due to increased link lengths 3 and 4.

Figure 11 shows an unfavorable timing ratio at R_{1min} which increases until R_{1max} . The plot demonstrates that the range of R_{1min} to R_{1max} can be reduced until the linkage has an acceptable value for the timing ratio. This results in a smaller arc of ground pivots and decreases the maximum displacement of the linkage.

All of these parameters are a function of links 3 and 4. As a result, there are two infinities of solutions to compare. Before this can be done, the transmission angles of the fourbar and the timing ratio must be controlled by reducing the range of R_{1min} and R_{1max} . Doing so will reduce the maximum displacement of the linkage but is necessary to increase performance of the linkage.

CONCLUSIONS

This paper describes a synthesis technique to design a linkage for a variable displacement piston pump/motor that can go to zero displacement and has a constant top dead center. Such a linkage is made by creating a base fourbar and attaching a slider to the end of link 4 with a link of equal length. By placing the slider at the ground pivot of link 4, it will exhibit zero displacement. If the ground link is moved along an arc about K_{TDC} , the slider will be displaced while returning to a constant TDC.

The results show that linkages can be built with a slider transmission angles greater than 65° and periods where the fourbar minimum transmission angle is greater than 45° . The footprint of variable linkages can be twice as large as that of a comparable fixed displacement crank slider, but the increase in size can be expected. Future work is planned to control the transmission angles and timing ratio and then to optimize the linkage over values of link lengths 3 and 4 in order to maximize transmission angles and displacement, minimize footprint area, and normalize the timing ratio. This work will help to create and analyze an efficient, variable displacement linkage which can be used in many applications

ACKNOWLEDGMENTS

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