Robust Multicast Beamforming for Spectrum Sharing-based Cognitive Radios

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***Abstract*—We consider a robust downlink beamforming optimization problem for secondary multicast transmission in a multiple-input multiple-output (MIMO) spectrum sharing cognitive radio (CR) network. The minimization of transmit power is formulated subject to both quality-of-service (QoS) constraints on the secondary receivers and interference temperature constraints on the primary users, under the assumption of imperfect channel state information (CSI). The problem is a non-convex quadratically constrained quadratic program (QCQP), and in general it is hard to achieve the global optimality. As a compromise, we present two randomized approximation algorithms for the problem via convex optimization techniques. Apart from the general setting of the robust beamforming problem, we identify one interesting special case, the robust problem of which can be solved efficiently. Simulation results are presented to demonstrate the performance gains of the proposed algorithms over an existing robust design.**

***Index Terms*—Robust multicast beamforming, MIMO cognitive radio networks, spectrum sharing, semidefinite programming relaxation, imperfect channel state information.**

I. Introduction

In cognitive radio (CR) networks, the idea of spectrum sharing using multiple transmit antennas has drawn much research interest recently. Spectrum sharing allows secondary and primary users to access the same channel simultaneously, by using the spatial degree of freedom at the secondary transmitter to avoid excessive interference to the primary users. For a comprehensive coverage of the recent advances, readers are referred to the magazine paper [1]; also [2], [3] for some recent specific works on CR transmit beamforming.

In this work we are interested in a robust multicast[1] transmit beamforming problem in the setting of a multiple-input multiple-output (MIMO) CR network, under the assumption of imperfect channel state information (CSI). A basic and meaningful problem formulation is to minimize transmit power subject to quality-of-service (QoS) constraints on the secondary receivers and interference temperature constraints (or termed as CR interference limiting constraints) on the primary receivers. To proceed, let us first discuss some related works. The 'primal' multicast transmit beamforming framework, i.e., that without CR and with perfect CSI, was originally developed in [4]; see also the survey paper [5]. In particular, that paper advocated to use semidefinite programming (SDP) relaxation to handle the multicast transmit optimization problems, an idea that has received growing attention recently. Its robust version under imperfect CSI was later studied in [6], where an interesting connection between non-robust and robust beamforming problems is revealed. More recently, the framework is extended to the CR scenario [3]. There, the robust CR multicast beamforming problem (our problem of interest) is also considered; the idea is to apply a conservative bound on both the QoS constraints and the interference suppressing constraints, thereby obtaining a quadratically constrained quadratic program (QCQP) formulation which is subsequently approximated by SDP relaxation.

In this paper, we propose two randomized approximation algorithms for the robust CR multicast downlink beamforming problem that can provide better approximation accuracies than the previous method [3]. Specifically, in one algorithm, we take into account an equivalent QCQP reformulation of the robust problem, of which we obtain a parameterized SDP relaxation problem. The parameterized SDP can be solved by searching a one-dimensional parameter over an interval, and a feasibility checking routine using SDP; a Gaussian randomization procedure is presented to give approximate solutions of the robust problem in a neat way, based on the solution of the parameterized SDP. In contrast, we herein improve both the problem reformulation (17) in [3] and the randomization procedure (cf. (30) in [3]), leading to better robust performance. In the other algorithm, we consider a convex (SDP) relaxation of the robust optimal beamforming problem resorting to S-lemma; we should note that S-lemma has been used in some other transmit beamforming contexts, e.g., [8], [9]. It turns out that the resulting SDP relaxation is looser than the previous parameterized SDP, giving rise to the possibility of returning lower transmit power at a small cost of feasibility rate.

In addition, we identify one particular, interesting, scenario, in which the global optimum of the robust beamforming problem can be found efficiently. The particular case is when there are "not too many" primary and secondary receivers involved, in which we show that the parameterized SDP relaxation is tight. The numerical simulation results show the outperformance of the proposed beamformers over the robust design in [3].

The paper is organized as follows. In Section II, we introduce the system model and formulate the robust optimal beamforming problem. In Section III, we propose the randomized approximation algorithms, and point out one solvable scenario of the robust beamforming problem. In Section IV, we present numerical examples showing the performance of three different algorithms. Finally, the paper is concluded in Section V.

II. System Model and Problem Formulation

Consider a CR network that has a secondary transmitter using N antennas to transmit common information to M secondary receivers, and that there are K primary users operating in the same spectrum. Let $\boldsymbol{H}_m \in \mathbb{C}^{N\times N_m}$ be the MIMO channel from the secondary transmitter to the mth secondary receiver, where the number of receive antennas is denoted by N_m. Similarly, let $\boldsymbol{G}_k \in \mathbb{C}^{N\times N'_k}$ be the MIMO channel from the secondary transmitter to the kth primary user, where the number of receive antennas is denoted by N'_k. The signal received by secondary receiver m is given by $\boldsymbol{x}_m(t) =$

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[1] Here multicast, in a physical layer sense, refers to common information broadcast to a multitude of intended receivers.

$\boldsymbol{H}_m^H \boldsymbol{y}(t)+\boldsymbol{n}_m(t)$, where $\boldsymbol{y}(t) \in \mathbb{C}^N$ is the secondary transmit signal vector, and $\boldsymbol{n}_m(t) \in \mathbb{C}^{N_m}$ is Gaussian noise vector, assumed to have zero mean and covariance $\sigma_m^2 \boldsymbol{I}$. [2] The secondary transmitter employs the multicast transmit beamforming scheme, in which we have $\boldsymbol{y}(t) = s(t)\boldsymbol{w}$ where $\boldsymbol{w} \in \mathbb{C}^N$ is the beamformer weight and $s(t) \in \mathbb{C}$ is the information signal. We assume $s(t)$ to be with zero mean and unit variance, without loss of generality. Moreover, assuming maximum ratio combining (MRC) receive beamforming for all the secondary receivers, the received SNR of secondary user m is $\mathsf{SNR}_m = \frac{\|\boldsymbol{H}_m^H \boldsymbol{w}\|^2}{\sigma_m^2}$, where $\|\cdot\|$ denotes the Euclidean norm or the Frobenius norm throughout the paper.

The multicast beamformer design may be formulated as (cf. [3], [5]):

$$\begin{aligned} \underset{\boldsymbol{w}}{\text{minimize}} \quad & \boldsymbol{w}^H \boldsymbol{w} \\ \text{subject to} \quad & \|\boldsymbol{H}_m^H \boldsymbol{w}\|^2 \geq \sigma_m^2 \tau_m, \quad m = 1, \ldots, M, \\ & \|\boldsymbol{G}_k^H \boldsymbol{w}\|^2 \leq \eta_k, \quad k = 1, \ldots, K, \end{aligned} \tag{1}$$

where τ_m specifies the minimal QoS of the secondary user m, in terms of SNR, and η_k specifies the maximal allowable interference level from the secondary transmitter to primary user k. Problem (1) has been considered in [3], where an effective approximation method called SDP relaxation has been applied.

Herein we consider the imperfect CSI case. In practice, one may not have perfect knowledge of the CSI especially for the links of primary users. The CSI errors may be caused by inaccurate channel estimation, quantization in channel feedback, and outdated CSI effects. Let $\boldsymbol{\Delta}_m \in \mathbb{C}^{N \times N_m}$ and $\boldsymbol{\Delta}'_k \in \mathbb{C}^{N \times N'_k}$ denote the CSI errors associated with $\boldsymbol{H}_m$ and $\boldsymbol{G}_k$, respectively. By assuming that the errors $\boldsymbol{\Delta}_m$ and $\boldsymbol{\Delta}'_k$ are deterministic norm-bounded, a worst-case robust version of (1) is given by (cf. problem (8) of [3]):

$$\underset{\boldsymbol{w}}{\min} \quad \boldsymbol{w}^H \boldsymbol{w} \tag{2a}$$

$$\text{s.t.} \quad \underset{\|\boldsymbol{\Delta}_m\| \leq \epsilon_m}{\min} \|(\boldsymbol{H}_m + \boldsymbol{\Delta}_m)^H \boldsymbol{w}\|^2 \geq \sigma_m^2 \tau_m, \forall m, \tag{2b}$$

$$\underset{\|\boldsymbol{\Delta}'_k\| \leq \epsilon'_k}{\max} \|(\boldsymbol{G}_k + \boldsymbol{\Delta}'_k)^H \boldsymbol{w}\|^2 \leq \eta_k, \forall k, \tag{2c}$$

where ϵ_m and ϵ'_k specify the bounds, or the worst-case magnitudes, of the CSI errors $\boldsymbol{\Delta}_m$ and $\boldsymbol{\Delta}'_k$, respectively. The robust beamforming problem (2) guarantees that for all admissible channel errors, all the secondary users must be served with QoSs no less than the specification $\{\tau_m\}$, and the interferences to all the primary users must be kept below $\{\eta_k\}$.

III. Efficient Algorithms for Robust Beamforming Problem

Robust beamforming problem (2) is hard to solve in general, due to its nature of non-convexity. In this section, we will propose two approximate solution schemes for the robust beamforming problem, and identify one interesting subclass of problem (2) that has a tight SDP relaxation. Thus that subclass of problem (2) can be solved in polynomial time, meaning that they are hidden convex programs. Let us start with an equivalent non-convex QCQP reformulation of (2).

[2]There are cases where interference from primary users to the secondary users contributes part of the noise terms $\boldsymbol{n}_m(t)$. While we may not physically model $\boldsymbol{n}_m(t)$ as being white in those cases (except for $N_m = 1$), we can transform the received model to an equivalent noise-white model by pre-whitening. To describe it, suppose that the noise $\boldsymbol{n}_m(t)$ has a positive definite, non-white, covariance $\boldsymbol{C}_m$. Let $\tilde{\boldsymbol{x}}_m(t) = \boldsymbol{C}_m^{-1/2} \boldsymbol{x}_m(t)$. We have $\tilde{\boldsymbol{x}}_m(t) = \tilde{\boldsymbol{H}}_m^H \boldsymbol{y}(t) + \tilde{\boldsymbol{n}}_m(t)$, where $\tilde{\boldsymbol{H}}_m = \boldsymbol{H}_m \boldsymbol{C}_m^{-1/2}$ is the transformed channel and $\tilde{\boldsymbol{n}}_m(t) = \boldsymbol{C}_m^{-1/2} \boldsymbol{n}_m(t)$ is white. Note that in this setup, the secondary receivers can simply send back the transformed channel state information $\tilde{\boldsymbol{H}}_m$ to the primary transmitter, rather than $\boldsymbol{H}_m, \boldsymbol{C}_m$ which incur a higher feedback overhead.

A. An equivalent QCQP reformulation of robust optimal beamforming problem (2)

Consider the first robust QoS constraint of (2b) in a slightly more general form, and set

$$f_1(\boldsymbol{w}) = \underset{\|\boldsymbol{E}_1^{1/2} \boldsymbol{\Delta}_1\| \leq \epsilon_1}{\text{minimize}} \|(\boldsymbol{H}_1 + \boldsymbol{\Delta}_1)^H \boldsymbol{w}\|^2 ,$$

where $\boldsymbol{E}_1 \succ \boldsymbol{0}$ governs the ellipsoid shape of the error set (or termed as the perturbation set in some robust optimization literature, e.g., [10]) and $\boldsymbol{w} \neq \boldsymbol{0}$. We claim that $f_1(\boldsymbol{w})$ has a closed-form expression as stated in the following lemma (see related results in [11], [12], [13]).

Lemma 3.1: It holds that

$$f_1(\boldsymbol{w}) = \left(\max\left\{ \|\boldsymbol{H}_1^H \boldsymbol{w}\| - \epsilon_1 \|\boldsymbol{E}_1^{-1/2} \boldsymbol{w}\|, 0 \right\} \right)^2 . \tag{3}$$

Proof: See Appendix A. ■

We remark that the optimal value $f_1(\boldsymbol{w})$ remains unchanged if $\|\boldsymbol{E}_1^{1/2} \boldsymbol{\Delta}_1\|$ is changed to the spectral norm (the maximal singular value) from the Frobenius norm. Like the proof in Lemma 3.1, the maximization problem in the first interference limiting constraint of (2c) has the optimal value $(\|\boldsymbol{G}_1^H \boldsymbol{w}\| + \epsilon'_1 \|\boldsymbol{w}\|)^2$. It thus follows that the robust beamforming problem (2) can be recast into

$$\underset{\boldsymbol{w}}{\min} \quad \boldsymbol{w}^H \boldsymbol{w} \tag{4a}$$

$$\text{s.t.} \quad \|\boldsymbol{H}_m^H \boldsymbol{w}\| \geq \sigma_m \sqrt{\tau_m} + \epsilon_m \|\boldsymbol{w}\|, \; m = 1, \ldots, M, \tag{4b}$$

$$\|\boldsymbol{G}_k^H \boldsymbol{w}\| \leq \sqrt{\eta_k} - \epsilon'_k \|\boldsymbol{w}\|, \; k = 1, \ldots, K. \tag{4c}$$

While problem (4) exhibits a similar form as (17) in [3], we would point out that problem (4) admits a larger feasible region (and thus gives lower transmission power) than (17) in [3]. To see this, we use the same configuration in [3], namely, setting $\boldsymbol{H}_m = \boldsymbol{h}_m$ and $\boldsymbol{G}_k = \boldsymbol{g}_k$ (i.e. $N_m = N'_k = 1, \forall m, k$). Hence, (4b) reduces to $(|\boldsymbol{h}_m^H \boldsymbol{w}| - \epsilon_m \|\boldsymbol{w}\|)^2 \geq \sigma_m^2 \tau_m, \forall m$. Observe that (17b) in [3] is given by

$$\boldsymbol{w}^H \tilde{\boldsymbol{H}}_m \boldsymbol{w} := \boldsymbol{w}^H \left(\boldsymbol{h}_m \boldsymbol{h}_m^H + \epsilon_m \left(\epsilon_m - 2\sqrt{\boldsymbol{h}_m^H \boldsymbol{h}_m} \right) \boldsymbol{I} \right) \boldsymbol{w} \geq \sigma_m^2 \tau_m,$$

$\forall m$. It is straightforward to check that $(|\boldsymbol{h}_m^H \boldsymbol{w}| - \epsilon_m \|\boldsymbol{w}\|)^2 \geq \boldsymbol{w}^H \tilde{\boldsymbol{H}}_m \boldsymbol{w}, \forall m$. This implies that any $\boldsymbol{w}$ fulfilling (17b) in [3] satisfies (4b). Similarly, one can show that any $\boldsymbol{w}$ satisfying (17c) in [3] fulfills (4c).

It is known that problem (4) is NP-hard [4] (in fact, problem (4) has been proved NP-hard, when $\epsilon_m = 0$, $\forall m$, and $\boldsymbol{G}_k = \boldsymbol{0}$, $\epsilon'_k = 0$, $\forall k$). Instead, one may resort to efficiently finding a suboptimal (or approximate) solution (e.g., see [3], [6], [4]).

In the following, we will propose randomized, SDP-based, methods for generating an approximate solution of the robust beamforming problem (2), as well as presenting some efficiently solvable scenario of problem (2).

B. A randomized approximation algorithm for the robust beamforming problem with ball perturbation

Problem (4) is tantamount to the following QCQP problem

$$\begin{aligned} \underset{\boldsymbol{w},\, t}{\text{minimize}} \quad & t^2 \\ \text{subject to} \quad & \|\boldsymbol{H}_m^H \boldsymbol{w}\| \geq \sigma_m \sqrt{\tau_m} + \epsilon_m t, \; m = 1, \ldots, M, \\ & \|\boldsymbol{G}_k^H \boldsymbol{w}\| \leq \sqrt{\eta_k} - \epsilon'_k t, \; k = 1, \ldots, K, \\ & \|\boldsymbol{w}\| = t. \end{aligned} \tag{5}$$

Note that any feasible point $(\boldsymbol{w}, t)$ of (5) must satisfy

$$\sqrt{\lambda_{\max}(\boldsymbol{H}_m \boldsymbol{H}_m^H)} \geq \frac{\|\boldsymbol{H}_m^H \boldsymbol{w}\|}{\|\boldsymbol{w}\|} \geq \frac{\sigma_m \sqrt{\tau_m}}{t} + \epsilon_m, \forall m \tag{6}$$

and

$$\sqrt{\lambda_{\min}(\boldsymbol{G}_k\boldsymbol{G}_k^H)} \le \frac{\|\boldsymbol{G}_k^H\boldsymbol{w}\|}{\|\boldsymbol{w}\|} \le \frac{\sqrt{\eta_k}}{t} - \epsilon_k', \ \forall k. \tag{7}$$

In fact, the first inequality in (6) follows from the basic property $\|\boldsymbol{w}\|^2\lambda_{\min}(\boldsymbol{A}) \le \boldsymbol{w}^H\boldsymbol{A}\boldsymbol{w} \le \|\boldsymbol{w}\|^2\lambda_{\max}(\boldsymbol{A})$ for a Hermitian matrix $\boldsymbol{A}$, and the second inequality in (6) is due to the feasibility in (5). Likewise, (7) is derived. Suppose that $\sqrt{\lambda_{\max}(\boldsymbol{H}_m\boldsymbol{H}_m^H) - \epsilon_m} > 0$ for all m (otherwise problem (5) would be infeasible). It follows that a necessary condition for t to be feasible for (5) is

$$t_0 \le t \le t_1, \tag{8}$$

where the lower bound t_0 and the upper bound t_1 are respectively given by

$$t_0 = \max_{1\le m\le M}\left\{\frac{\sigma_m\sqrt{\tau_m}}{\sqrt{\lambda_{\max}(\boldsymbol{H}_m\boldsymbol{H}_m^H) - \epsilon_m}}\right\} \tag{9}$$

and

$$t_1 = \min_{1\le k\le K}\left\{\frac{\sqrt{\eta_k}}{\sqrt{\lambda_{\min}(\boldsymbol{G}_k\boldsymbol{G}_k^H)} + \epsilon_k'}\right\}. \tag{10}$$

Then problem (5) indeed amounts to the following problem

$$\begin{aligned}
&\min_{\boldsymbol{W},\, t} \quad t && \text{(11a)}\\
&\text{s.t.} \quad \boldsymbol{H}_m\boldsymbol{H}_m^H \bullet \boldsymbol{W} \ge (\sigma_m\sqrt{\tau_m} + \epsilon_m t)^2,\ m = 1,\dots,M, && \text{(11b)}\\
&\qquad \boldsymbol{G}_k\boldsymbol{G}_k^H \bullet \boldsymbol{W} \le (\sqrt{\eta_k} - \epsilon_k' t)^2,\ k = 1,\dots,K, && \text{(11c)}\\
&\qquad \boldsymbol{I} \bullet \boldsymbol{W} = t^2, && \text{(11d)}\\
&\qquad \boldsymbol{W} \succeq \boldsymbol{0},\ \text{Rank}\,(\boldsymbol{W}) = 1,\ t_0 \le t \le t_1, && \text{(11e)}
\end{aligned}$$

where $\boldsymbol{A} \bullet \boldsymbol{B} = \text{tr}\,(\boldsymbol{A}\boldsymbol{B}^H)$. Dropping the rank-one constraint yields the relaxation problem

$$\begin{aligned}
&\underset{\boldsymbol{W},\, t}{\text{minimize}} && t\\
&\text{subject to} && \text{(11b), (11c), (11d) satisfied,}\\
& && \boldsymbol{W} \succeq \boldsymbol{0},\ t_0 \le t \le t_1.
\end{aligned} \tag{12}$$

Fixing t, problem (12) is an SDP feasibility problem. Now, let $g(t)$ be the optimal value of such a feasibility problem. In other words, we have $g(t) = t$ if (12) is feasible for a given t (any feasible $\boldsymbol{W}$ is thus optimal), and $g(t) = +\infty$ if it is infeasible at point t. Therefore, (12) amounts to the one-dimensional optimization problem:

$$\begin{aligned}
&\underset{t}{\text{minimize}} && g(t)\\
&\text{subject to} && t_0 \le t \le t_1.
\end{aligned} \tag{13}$$

In other words, (12) can be solved by solving (13): fixing t, solving the SDP feasibility problem (obtaining $g(t)$), and reducing t iteratively. In the optimization literature there are some derivative-free methods for solving the one-dimensional optimization problem (13). One of these methods is called compass or coordinate search (cf. [14, Algorithm 3.1 and Section 8.1], [15, Algorithm 7.1]). In practice, we adopt either the uniform sampling or the Matlab function `fminbnd`, in order to output a satisfactory solution.

Once such a solution $(\boldsymbol{W}^\star, t^\star)$ of (12) is obtained, we retrieve a rank-one approximate solution of (12) by making use of $\boldsymbol{W}^\star$. Particularly, a randomization procedure is proposed as follows: Take random vectors $\boldsymbol{w}_i$, $i = 1,\dots,I$, from the complex normal distribution $\mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{W}^\star)$, and compute

$$\lambda(\boldsymbol{w}_i) \;=\; \min\left\{\frac{\|\boldsymbol{H}_m^H\boldsymbol{w}_i\| - \sigma_m\sqrt{\tau_m}}{\epsilon_m}, \frac{\sqrt{\eta_k} - \|\boldsymbol{G}_k^H\boldsymbol{w}_i\|}{\epsilon_k'},\right. \\ \left. m = 1,\dots,M,\ k = 1,\dots,K\right\}. \tag{14}$$

Clearly, if $\|\boldsymbol{w}_i\| \le \lambda(\boldsymbol{w}_i)$, then $(\boldsymbol{w}_i, \|\boldsymbol{w}_i\|)$ is feasible for (5).

Summarizing, a randomized approximate solution of problem (4) (or equivalently (5)) can be generated by Algorithm 1.

Algorithm 1 Gaussian randomization procedure for robust beamforming problem (4)

Input: $\boldsymbol{H}_m$, $\boldsymbol{G}_k$, σ_m, τ_m, ϵ_m, η_k, ϵ_k', I;

Output: a randomized approximate solution $\boldsymbol{w}$ of (4);

1: solve (13), and find an optimal solution $(\boldsymbol{W}^\star, t^\star)$;

2: if Rank $(\boldsymbol{W}^\star) = 1$, then output $\boldsymbol{w}^\star$ with $\boldsymbol{w}^\star\boldsymbol{w}^{\star H} = \boldsymbol{W}^\star$ and terminate;

3: draw random vectors $\boldsymbol{w}_i \in \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{W}^\star)$, $i = 1,\dots,I$, and compute $\lambda(\boldsymbol{w}_i)$ by (14);

4: pick up $\boldsymbol{w}_{i_0}$ such that $i_0 = \arg\min\{\|\boldsymbol{w}_i\| : \|\boldsymbol{w}_i\| \le \lambda(\boldsymbol{w}_i), i = 1,\dots,I\}$.

C. A convex relaxation for robust beamforming problem (2) by S-lemma

In this subsection, we study an SDP relaxation of the robust beamforming problem (2), resorting to S-lemma (e.g., see [10, Appendix B.2]). Let $\boldsymbol{\delta}_m = \text{vec}\,(\boldsymbol{\Delta}_m)$, $\boldsymbol{h}_m = \text{vec}\,(\boldsymbol{H}_m)$, and $\boldsymbol{g}_k$ and $\boldsymbol{\delta}_k'$ are defined similarly. Problem (2) can be reformulated equivalently into the following problem

$$\begin{aligned}
\min_{\boldsymbol{w}} \quad & \boldsymbol{I} \bullet \boldsymbol{W}\\
\text{s.t.} \quad & \boldsymbol{\delta}_m^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{\delta}_m + 2\Re(\boldsymbol{h}_m^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{\delta}_m) + \boldsymbol{h}_m^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{h}_m\\
& \qquad \ge \sigma_m^2\tau_m, \forall\|\boldsymbol{\delta}_m\|^2 \le \epsilon_m^2, \forall m,\\
& (\boldsymbol{\delta}_k')^H(\boldsymbol{I}\otimes\boldsymbol{W})(\boldsymbol{\delta}_k') + 2\Re(\boldsymbol{g}_k^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{\delta}_k') +\\
& \qquad \boldsymbol{g}_k^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{g}_k \le \eta_k, \forall\|\boldsymbol{\delta}_k'\|^2 \le (\epsilon_k')^2, \forall k,\\
& \boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H.
\end{aligned} \tag{15}$$

Here we use the fact that $\text{tr}\,(\boldsymbol{A}^H\boldsymbol{B}\boldsymbol{C}) = \text{vec}\,(\boldsymbol{A})^H(\boldsymbol{I}\otimes\boldsymbol{B})\text{vec}\,(\boldsymbol{C})$, and ignore the dimension of each $\boldsymbol{I}$ since the dimension is clear in the context. The so-called S-lemma (e.g., see [10]) says that the condition

$$\boldsymbol{x}^H\boldsymbol{B}\boldsymbol{x} + 2\Re(\boldsymbol{b}^H\boldsymbol{x}) + \beta \ge 0, \forall\, \boldsymbol{x}^H\boldsymbol{A}\boldsymbol{x} + \alpha \ge 0$$

is equivalent to the linear matrix inequality (LMI):

$$\exists\lambda \ge 0 : \begin{bmatrix} \boldsymbol{B} - \lambda\boldsymbol{A} & \boldsymbol{b}\\ \boldsymbol{b}^H & \beta - \lambda\alpha \end{bmatrix} \succeq \boldsymbol{0},$$

provided that some Slater condition holds. By applying S-lemma to the constraints of (15) (e.g., for the first constraint of (15), setting $\boldsymbol{B} = (\boldsymbol{I}\otimes\boldsymbol{W})$, $\boldsymbol{b}^H = \boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{W})$, $\beta = \boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{h}_1 - \sigma_1^2\tau_1$, $\boldsymbol{A} = -\boldsymbol{I}$, $\alpha = \epsilon_1^2$), we can reexpress the constraints respectively into LMI constraints; therefore, (15) is reformulated to the relaxed problem (removing the rank-one constraint on $\boldsymbol{W}$):

$$\begin{aligned}
&\underset{\boldsymbol{w}, \{\mu_m\}, \{\lambda_k\}}{\text{minimize}} && \boldsymbol{I} \bullet \boldsymbol{W} && \text{(16a)}\\
&\text{subject to} && \begin{bmatrix} \boldsymbol{I}\otimes\boldsymbol{W} + \mu_m\boldsymbol{I} & (\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{h}_m\\ \boldsymbol{h}_m^H(\boldsymbol{I}\otimes\boldsymbol{W}) & \boldsymbol{h}_m^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{h}_m - \sigma_m^2\tau_m - \mu_m\epsilon_m^2 \end{bmatrix}\\
& && \qquad \succeq \boldsymbol{0}, m = 1,\dots,M, && \text{(16b)}\\
& && \begin{bmatrix} \boldsymbol{I}\otimes\boldsymbol{W} + \lambda_k\boldsymbol{I} & (\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{g}_k\\ \boldsymbol{g}_k^H(\boldsymbol{I}\otimes\boldsymbol{W}) & \boldsymbol{g}_k^H(\boldsymbol{I}\otimes\boldsymbol{W})\boldsymbol{g}_k - \eta_k - \lambda_k(\epsilon_k')^2 \end{bmatrix}\\
& && \qquad \preceq \boldsymbol{0}, k = 1,\dots,K, && \text{(16c)}\\
& && \boldsymbol{W} \succeq \boldsymbol{0},\ \mu_m \ge 0,\ \forall m,\ \lambda_k \le 0,\ \forall k. && \text{(16d)}
\end{aligned}$$

Note that (16) is a convex relaxation of (2), while problem (12) is a non-convex relaxation of problem (4) (or equivalently, (2)). We have shown in the last subsection that even though (12) is not a convex optimization problem, the global optimality of it may be achieved by solving a one-dimensional optimization problem over an interval. To discuss some relations between the two relaxations, we have the following observations.

Proposition 3.2: It holds that

1) if $(\boldsymbol{W}, \sqrt{\boldsymbol{I} \bullet \boldsymbol{W}})$ is feasible for (12), then $\boldsymbol{W}$, together with some $\mu_m \geq 0$ and $\lambda_k \leq 0$, is feasible for (16);
2) if $(\boldsymbol{w}\boldsymbol{w}^H, \{\mu_m\}, \{\lambda_k\})$ is feasible for (16), then $(\boldsymbol{w}\boldsymbol{w}^H, \|\boldsymbol{w}\|)$ is feasible for (12).

Proof: See Appendix B. ■

The proposition indicates that the convex relaxation (16) is not as tight as the non-convex relaxation (12) in general (namely, (16) always gives a lower bound of (12)). As to the second argument of the proposition, we provide an alternative proof without using Lemma 3.1 and S-lemma, notwithstanding the fact that if (16) has an optimal rank-one solution $\boldsymbol{w}\boldsymbol{w}^H$, then problems (16) and (2) are equivalent due to S-lemma, and problems (2) and (12) are equivalent due to Lemma 3.1.

Comparing to (12), SDP problem (16) is solved in a single step (unlike (12) resorting to solving one-dimension optimization problem (13)), and we take advantage of it to solve the robust beamforming problem (2). Precisely, the advantages of the relaxation (16) are three-fold: (i) if we get a rank-one optimal solution $\boldsymbol{w}^\star \boldsymbol{w}^{\star H}$ (16), then there is no gap between (2) and (16), and thus we do not have to iteratively solve (13); (ii) in case of getting a solution $\boldsymbol{W}^\star$ of rank two or higher for (16), the optimal value of (16) can serve as a new t_0 for solving (13); namely, update $t_0 := \max\{t_0, \sqrt{\boldsymbol{I} \bullet \boldsymbol{W}^\star}\}$; (iii) it is possible to use $\boldsymbol{W}^\star$ as a covariance in order to get an approximate solution for (4), according to (14). We point out that the observation in the third point is interesting. In fact, it is not obvious how to generate, from $\boldsymbol{W}^\star$, a beamforming vector $\boldsymbol{w}$ so that $\boldsymbol{w}\boldsymbol{w}^H$ satisfies the SDP constraints (16b), (16c). Nevertheless, according to (14), the generated beamforming vector $\boldsymbol{w}$ from $\boldsymbol{W}^\star$ fulfills (4b), (4c), thus satisfies (2b), (2c), which in turn implies that $\boldsymbol{w}\boldsymbol{w}^H$ is feasible for (16b), (16c). As a consequence, we conclude that the randomized approximation algorithm for (4) via the SDP relaxation (16) consists of solving the SDP (16) (obtaining an optimal solution $(\boldsymbol{W}^\star, \{\mu_m^\star\}, \{\lambda_k^\star\})$), and steps 2-4 of Algorithm 1.

Note that the complexities of two approximation algorithms are dominated by solving the respective SDP relaxation problems. In Algorithm 1, the cost of outputting a solution by t-search is about 20 times empirically of solving an SDP feasibility problem, which has worst-case complexity of $O((\max\{M+K, N\})^4 N^{0.5} \log(1/\zeta))$ for a given accuracy ζ (see [7]); in the algorithm via (16), the computational cost is higher since the sizes of the involved SDP cones are quite large; for instance, in the particular case of $N_m = N_k' = 1$ $\forall m, k$, the complexity is up to of $O(N^{6.5} \log(1/\zeta))$ for a small $(M+K)$ (see [16]).

D. Solvable subclass of robust beamforming problem via SDP relaxation

In this subsection, we shall elaborate that robust beamforming problem (2) can be solved efficiently with parameters such that $M+K=3$ and $N \geq 3$ (i.e., the number of primary and secondary receivers equals three and the number of the transmit antennas is not less than three), or with parameter condition $M+K=2$.

Let $t^\star$ be a numerical minimizer for $g(t)$ over the interval $[t_0, t_1]$ as in problem (13), and $\boldsymbol{W}^\star$ be a corresponding feasible solution, namely, $(\boldsymbol{W}^\star, t^\star)$ complies with the constraints of (12). To proceed, let us assume $M=2$ and $K=1$ without loss of generality. Then, it follows that

$$\boldsymbol{H}_m \boldsymbol{H}_m^H \bullet \boldsymbol{W}^\star \geq (\delta_m \sqrt{\tau_m} + \epsilon_m t^\star)^2,\ m = 1, 2,$$

$$\boldsymbol{G}_1 \boldsymbol{G}_1^H \bullet \boldsymbol{W}^\star \leq (\eta_1 - \epsilon_1' t^\star)^2,\ \boldsymbol{I} \bullet \boldsymbol{W}^\star = (t^\star)^2.$$

It is verified immediately that the conditions of the specific rank-one matrix decomposition theorem 2.3 of [17] are satisfied, and thus one is able to polynomially construct a matrix $\boldsymbol{w}^\star \boldsymbol{w}^{\star H}$ according to the rank-one decomposition theorem, such that

$$\boldsymbol{w}^{\star H} \boldsymbol{H}_m \boldsymbol{H}_m^H \boldsymbol{w}^\star = \boldsymbol{H}_m \boldsymbol{H}_m^H \bullet \boldsymbol{W}^\star,\ m = 1, 2,$$
$$\boldsymbol{w}^{\star H} \boldsymbol{G}_1 \boldsymbol{G}_1^H \boldsymbol{w}^\star = \boldsymbol{G}_1 \boldsymbol{G}_1^H \bullet \boldsymbol{W}^\star,\ \|\boldsymbol{w}^\star\|^2 = \boldsymbol{I} \bullet \boldsymbol{W}^\star.$$

This implies that $(\boldsymbol{w}^\star \boldsymbol{w}^{\star H}, t^\star)$ is feasible for (12); thus $(\boldsymbol{w}^\star, t^\star)$ is feasible for (5). Therefore, we conclude that $\boldsymbol{w}^\star$ is optimal for (5) since the problem shares the same optimal value $t^\star$ with its relaxation problem (12). For the scenario with parameters fulfilling $M+K=2$, it follows from the specific rank-one theorem of [18] that a rank-one matrix $\boldsymbol{w}\boldsymbol{w}^H$ can be discovered efficiently so that

$$(\boldsymbol{H}_1 \boldsymbol{H}_1^H - \frac{(\delta_1 \sqrt{\tau_1} + \epsilon_1 t^\star)^2}{(t^\star)^2} \boldsymbol{I}) \bullet \boldsymbol{w}\boldsymbol{w}^H = 0,$$

$$(\boldsymbol{G}_1 \boldsymbol{G}_1^H - \frac{(\eta_1 - \epsilon_1' t^\star)^2}{(t^\star)^2} \boldsymbol{I}) \bullet \boldsymbol{w}\boldsymbol{w}^H = 0.$$

Then, it is seen that $\boldsymbol{w}^\star = \frac{t^\star}{\|\boldsymbol{w}\|} \boldsymbol{w}$ is feasible for (5), and the objective function value is the same as the optimal value of its relaxation problem (12), whence $\boldsymbol{w}^\star$ is optimal.

IV. Simulation results

We consider a CR network with a three-antenna secondary transmitter, five single-antenna secondary receivers and two single-antenna primary receivers (i.e., $N=3$, $M=5$, $K=2$, and $N_m = N_k' = 1$, $\forall m, k$). The elements of the channels (from the secondary transmitter to either the primary or the secondary users) are assumed to be i.i.d. complex Gaussian distributed with mean 0 and variance 1. We fix the secondary receivers' noise variance $\sigma_m^2 = 1$ $\forall m$.

Fig. 1 (a) examines how the average transmit power is affected by the radius of the channel perturbation set. In the simulation, we set $\tau_m = 10$ dB $\forall m$ and $\eta_k = 0$ dB $\forall k$. The same channel perturbation level is assumed for all primary and secondary channels, i.e., $\epsilon_m = \epsilon_k' = \epsilon$, $\forall m, k$. A total of 3000 channel realizations (each with 10000 Gaussian randomizations) are tested. We compare our two proposed robust beamforming designs, namely t-search (problem (5)) and S-lemma (problem (16)) designs, with an existing robust design provided by problem (17) of [3]. Moreover, as the robust power minimization problem with QoS constraints could be intrinsically infeasible, the average transmit power in Fig. 1 (a) is obtained by averaging only those channel realizations for which all the three robust designs are feasible, i.e., at least one feasible beamforming solution can be found for each design after randomization procedure. In the legend, the "beamformer" stands for the result after randomization, while "SDP relaxation value" means the optimal value of the SDP relaxations corresponding to the three robust designs. A practically logical result we see from Fig. 1 (a) is that higher transmit power is required to assure larger radius of the channel error set (i.e., provide more robust beamformer). Fig. 1 (a) also shows that the average transmit powers by our proposed robust beamformers are lower than that by (17) of [3] in general. This means that the former methods are less conservative than the latter. Let us compare the performance of our two robust proposed designs. In Fig. 1 (a), for the SDP relaxation values, we note that S-lemma yields a slightly lower value than t-search, which is consistent with our claim in Prop. 3.2, i.e., the relaxation (16) (S-lemma) is looser than (12) (t-search). For the beamformer's power, we see that S-lemma leads to slightly better performance than t-search. As observed, the performance gap of our two algorithms however is not big. This phenomenon may be caused by the precision of the relaxed solution, the approximation procedure employed and the simulation settings. To get a better understanding of the conservativeness, Fig. 1 (b) plots the feasibility rate of the three designs with the same setup as Fig. 1 (a). Here the feasibility rate is denoted as the ratio of the number of channel realizations, for which we can generate a feasible beamformer via randomization, over the total 3000 channel trials. It can be seen from Fig. 1 (b) that the proposed two robust designs have much higher feasible rates than that of [3] over the whole perturbation radii tested. In Fig. 1 (b), we also observe that t-search method yields slightly higher feasibility rate than S-lemma. In contrast, as Fig. 1 (a) shows, S-lemma design has superior performance in terms of the SDP relaxation values and the transmit power of beamformers. In other words, there is a trade-off between the two proposed robust designs.

Fig. 2 investigates how the average transmit power changes with the QoS of the secondary users and the interference level requirement of the primary users. We fix $\epsilon_m = \epsilon'_k = 0.04$, $\forall k, m$, $\tau_m = 10\,\text{dB}$, $\forall m$, and $\eta_k = 0\,\text{dB}\,\forall k$, if not mentioned. Fig. 2 (a) plots the average transmit power versus the QoS of the secondary users of the two proposed robust designs. To provide a reference for the transmit power lower bound, we also plot the result of perfect CSI (i.e., $\epsilon_m = \epsilon'_k = 0$, $\forall k, m$). It can be seen that as τ increase, we need to use more transmit power to support higher secondary users' QoS requirements. Also there is marginal performance difference between the t-search and the S-lemma based robust designs. Fig. 2 (b) shows the average transmit power versus the primary users' interference level requirement. As expected, a loose interference temperature requirement leads to low average transmit power, and vice versa.

V. Conclusion

We have considered a robust secondary multicast beamformer design problem for spectrum sharing in a MIMO CR network. Two efficient algorithms for the robust problem have been proposed: one algorithm includes solving a one-dimensional optimization problem, checking the feasibility of SDPs, and a Gaussian randomized procedure; the other algorithm resorts to S-lemma leading to an SDP relaxation problem and a randomization procedure. In the special case of "not too many" primary and secondary receivers (cf. Sec. III-D), we have also proved that the robust optimal beamforming problem can be solved efficiently. The performance of the proposed beamforming designs has been demonstrated by simulations. Future research topics may include efficiently robust design of multi-group and multicast beamformer for a CR network.

Appendix

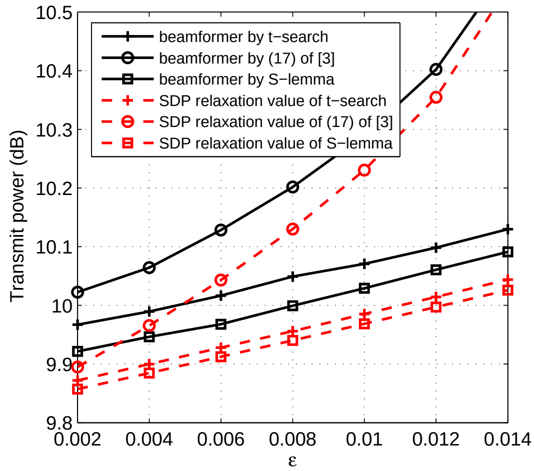
A. Proof of Lemma 3.1

Proof: When $\|\boldsymbol{H}_1^H\boldsymbol{w}\| \leq \epsilon_1\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\|$, we select $\boldsymbol{\Delta}_1 = -\frac{\boldsymbol{E}_1^{-1}\boldsymbol{w}\boldsymbol{w}^H\boldsymbol{H}_1}{\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\|^2}$. It is easily verified that $\|\boldsymbol{E}_1^{1/2}\boldsymbol{\Delta}_1\| = \|\boldsymbol{H}_1^H\boldsymbol{w}\|/\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\| \leq \epsilon_1$ and $\|(\boldsymbol{H}_1+\boldsymbol{\Delta}_1)^H\boldsymbol{w}\| = 0$.

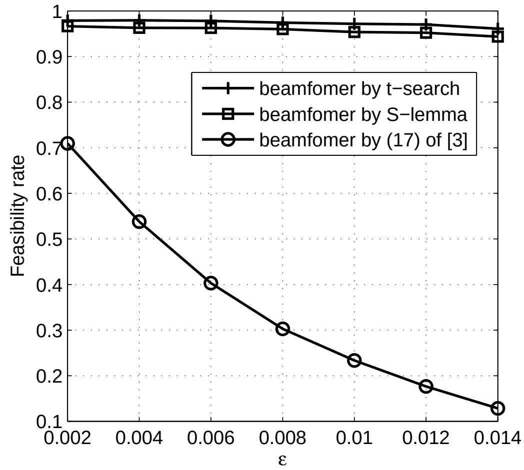
Suppose $\|\boldsymbol{H}_1^H\boldsymbol{w}\| > \epsilon_1\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\|$. It then follows that

$$\begin{aligned}
\|(\boldsymbol{H}_1+\boldsymbol{\Delta}_1)^H\boldsymbol{w}\| &\geq \|\boldsymbol{H}_1^H\boldsymbol{w}\| - \|(\boldsymbol{E}_1^{1/2}\boldsymbol{\Delta}_1)^H\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\| \\
&\geq \|\boldsymbol{H}_1^H\boldsymbol{w}\| - \|\boldsymbol{E}_1^{1/2}\boldsymbol{\Delta}_1\|\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\| \\
&\geq \|\boldsymbol{H}_1^H\boldsymbol{w}\| - \epsilon_1\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\| > 0,
\end{aligned}$$

and the inequality chain become equality chain when $\boldsymbol{\Delta}_1 = -\epsilon_1\boldsymbol{E}_1^{-1}\boldsymbol{w}\boldsymbol{w}^H\boldsymbol{H}_1/(\|\boldsymbol{H}_1^H\boldsymbol{w}\|\|\boldsymbol{E}_1^{-1/2}\boldsymbol{w}\|)$ (it is seen that $\|\boldsymbol{E}^{1/2}\boldsymbol{\Delta}_1\| = \epsilon_1$). ■



(a)



(b)

Fig. 1. (a) Average transmit power versus the radius of channel perturbation set. (b) Feasibility rate versus perturbation radius ϵ.

B. Proof of Proposition 3.2

Proof: (1) Since $(\boldsymbol{W}, \sqrt{\boldsymbol{I} \bullet \boldsymbol{W}})$ is feasible for (12), hence $\boldsymbol{H}_m\boldsymbol{H}_m^H \bullet \boldsymbol{W} \geq (\sigma_m\sqrt{\tau_m} + \epsilon_m\sqrt{\boldsymbol{I}\bullet\boldsymbol{W}})^2$, $\forall m$, which means that

$$\|\boldsymbol{W}^{1/2}\boldsymbol{H}_m\| - \epsilon_m\|\boldsymbol{W}^{1/2}\| \geq \sigma_m\sqrt{\tau_m},\ \forall m. \tag{17}$$

Likewise we have $\|\boldsymbol{W}^{1/2}\boldsymbol{G}_k\| + \epsilon'_k\|\boldsymbol{W}^{1/2}\| \leq \sqrt{\eta_k}$, $\forall k$. Observe that $\|\boldsymbol{W}^{1/2}(\boldsymbol{H}_m + \boldsymbol{\Delta}_m)\| \geq \|\boldsymbol{W}^{1/2}\boldsymbol{H}_m\| - \|\boldsymbol{W}^{1/2}\boldsymbol{\Delta}_m\| \geq \|\boldsymbol{W}^{1/2}\boldsymbol{H}_m\| - \epsilon_m\|\boldsymbol{W}^{1/2}\|$, for $\boldsymbol{\Delta}_m : \|\boldsymbol{\Delta}_m\| \leq \epsilon_m$. Therefore, it follows from (17) that $\sigma_m^2\tau_m \leq \min_{\|\boldsymbol{\Delta}_m\|\leq\epsilon_m} \text{tr}\,((\boldsymbol{H}_m + \boldsymbol{\Delta}_m)^H\boldsymbol{W}(\boldsymbol{H}_m + \boldsymbol{\Delta}_m))$. Similarly, it has $\eta_k \geq \max_{\|\boldsymbol{\Delta}'_k\|\leq\epsilon'_k} \text{tr}\,(\boldsymbol{G}_k + \boldsymbol{\Delta}'_k)^H\boldsymbol{W}(\boldsymbol{G}_k + \boldsymbol{\Delta}'_k)$. By S-lemma, we conclude that $(\boldsymbol{W}, \{\mu_m\}, \{\lambda_k\})$ is feasible for (16) for some $\mu_m \geq 0$ and some $\lambda_k \leq 0$, $m = 1, \ldots, M$ and $k = 1, \ldots, K$.

(2) Let us re-denote $\boldsymbol{\delta}_m = \text{vec}\,(\boldsymbol{\Delta}_m)$, $\boldsymbol{h}_m = \text{vec}\,(\boldsymbol{H}_m)$; $\boldsymbol{\delta}'_k$ and $\boldsymbol{g}_k$ are defined analogously. Suppose that $(\boldsymbol{w}\boldsymbol{w}^H, \{\mu_m\}, \{\lambda_k\})$ is feasible for (16). Let us look into the first constraint. Suppose that $\mu_1 > 0$. It follows from the first constraint of (16) and Schur complement lemma that

$$\begin{aligned}
&\boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)\boldsymbol{h}_1 - \sigma_1^2\tau_1 - \mu_1\epsilon_1^2 \geq \\
&\quad \boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H + \mu_1\boldsymbol{I})^{-1}(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)\boldsymbol{h}_1.
\end{aligned} \tag{18}$$

It is straightforward to verify that $(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H + \mu_1\boldsymbol{I})^{-1} = \frac{1}{\mu_1}(\boldsymbol{I} - \frac{\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H}{\mu_1+\|\boldsymbol{w}\|^2})$ by noting that $(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H) = \|\boldsymbol{w}\|^2(\boldsymbol{I}\otimes$

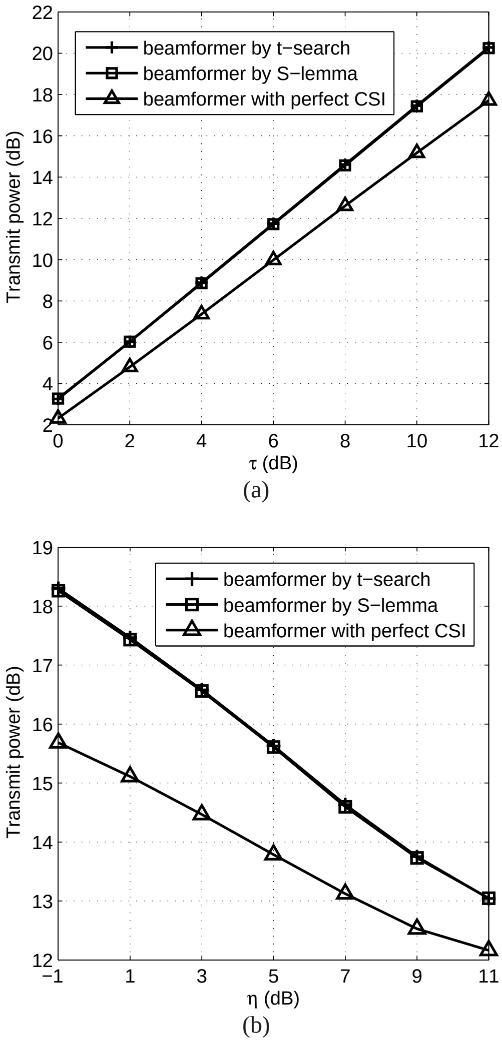


Fig. 2. (a) Average transmit power versus the QoS of the secondary users. (b) Average transmit power versus the interference level requirement of the primary users.

$\boldsymbol{w}\boldsymbol{w}^H$), and to check that the right-hand side of (18) is equal to $\frac{\|\boldsymbol{w}\|^2}{\mu_1+\|\boldsymbol{w}\|^2}\|(\boldsymbol{I}\otimes\boldsymbol{w}^H)\boldsymbol{h}_1\|^2$. It follows from (18) that

$$\begin{aligned}\|(\boldsymbol{I}\otimes\boldsymbol{w}^H)\boldsymbol{h}_1\|^2 &\geq (1+\frac{\|\boldsymbol{w}\|^2}{\mu_1})(\sigma_1^2\tau_1+\mu_1\epsilon_1^2)\\ &= \epsilon_1^2\|\boldsymbol{w}\|^2+\sigma_1^2\tau_1+\frac{\|\boldsymbol{w}\|^2\sigma_1^2\tau_1}{\mu_1}+\mu_1\epsilon_1^2\\ &\geq \epsilon_1^2\|\boldsymbol{w}\|^2+\sigma_1^2\tau_1+2\|\boldsymbol{w}\|\sigma_1\epsilon_1\sqrt{\tau_1},\end{aligned}$$

which is equivalent to $\boldsymbol{H}_1\boldsymbol{H}_1^H\bullet\boldsymbol{w}\boldsymbol{w}^H\geq(\sigma_1\sqrt{\tau_1}+\epsilon_1\sqrt{\boldsymbol{I}\bullet\boldsymbol{w}\boldsymbol{w}^H})^2$. In words, for any $\mu_1>0$, we have

$$\left\{\boldsymbol{w}:\begin{bmatrix}\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H+\mu_1\boldsymbol{I} & (\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)\boldsymbol{h}_1\\ \boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H) & \boldsymbol{h}_1^H(\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H)\boldsymbol{h}_1-\sigma_1^2\tau_1-\mu_1\epsilon_1^2\end{bmatrix}\right.$$
$$\left.\succeq\boldsymbol{0}\right\}\subseteq\{\boldsymbol{w}:\boldsymbol{H}_1\boldsymbol{H}_1^H\bullet\boldsymbol{w}\boldsymbol{w}^H\geq(\sigma_1\sqrt{\tau_1}+\epsilon_1\sqrt{\boldsymbol{I}\bullet\boldsymbol{w}\boldsymbol{w}^H})^2\}.$$

By a limiting argument, the inclusion relation still holds for $\mu_1=0$. This means that $\boldsymbol{w}\boldsymbol{w}^H$ fulfills the first constraint of (12). Similarly, we can show that $\boldsymbol{w}\boldsymbol{w}^H$ also fulfills the second to the M-th constraints.

Now let us deal with the $(M+1)$-th constraints of (16). Due to the feasibility of $(\boldsymbol{w}\boldsymbol{w}^H,\{\mu_m\},\{\lambda_k\})$, we see that $\boldsymbol{I}\otimes\boldsymbol{w}\boldsymbol{w}^H+\lambda_1\boldsymbol{I}\preceq\boldsymbol{0}$, which means $\boldsymbol{w}\boldsymbol{w}^H+\lambda_1\boldsymbol{I}\preceq\boldsymbol{0}$, which in turn implies $\lambda_1\leq-\|\boldsymbol{w}\|^2$. In a similar way, we can show that $\boldsymbol{w}\boldsymbol{w}^H$ satisfies the second set of constraints of (12). ∎

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